Central Bank Digital Currency: A Corporate Finance Perspective*

Mei Dong† Sylvia Xiaolin Xiao‡

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Abstract

We build models with an interest-bearing central bank digital currency (CBDC) to investigate the impacts of issuing CBDC on banking and the macroeconomy. From a corporate finance perspective, we find various designs of CBDC lead to different impacts. The design issues considered include: (1) a universal CBDC system or a system with CBDC and cash coexisting; (2) CBDC being a complement or substitute to bank deposits; and (3) banks having access to CBDC or not. As for policy, we find the interest rate of CBDC can be an effective policy tool, which has pass-though effects to the real deposit and loan rates, firm investment and the real economy. In addition, negative interest rates can be an option for policy makers, particularly in the economy with a universal CBDC.

Key words: CBDC, Corporate Finance, Banking, Negative Interest Rate

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†Department of Economics, University of Melbourne. Email: mei.dong@unimelb.edu.au

‡Department of Applied Economics, Guanghua School of Management, Peking University. Email: sylvia.xiao@gsm.pku.edu.cn

1 Introduction

The current fiat (paper) money system has been challenged by private cryptocurrencies since Bitcoin was created in 2009, based on a blockchain technology. This has pushed policy makers and researchers to study the possibility of introducing a central bank digital currency (CBDC) in recent years. Indeed, the blockchain technology may bring a revolution to the current financial system and central banking, since it can support decentralized payment without the need to designate a third-party that controls the currency or payment network. Despite their huge price volatilities, Bitcoin-like cryptocurrencies have gained great popularity among individual and institutional investors. By June 14, 2019, there were 2236 types of cryptocurrencies in total (still growing), and the market value of Bitcoin (top 1 cryptocurrency) and Ethereum (top 2) was US$ 147 billion and US$ 27 billion, respectively.

More importantly, on June 18, 2019, Facebook and its partners issued the white paper of "Libra", which is a new cryptocurrency with the mission "to enable a simple global currency and financial infrastructure that empowers" over 2.7 billion Facebook users. It will still use the blockchain technology, but the design is to be a "stablecoin" which aims to minimize the price volatility, with the full backup of reserves from a basket of multiple fiat currencies and credible government securities. Compared to Bitcoin-like cryptocurrencies, these features make Libra more possible to serve as a "currency", i.e., serving as a medium of exchange, a unit of account and a store of value. Therefore, the news of Libra has caused huge shocks and concerns among central banks and financial regulators all over the world. On June 30, 2019, Agustin Carstens, the general manager of BIS, urged that "central banks may have to issue their own digital currencies *sooner than expected*" (FT, 2019). In China, official news from PBC (People's Bank of China) confirms that the State Council of China has formally approved the plan of central bank digital currency (July 8, 2019, China ORG website). And the Chinese version of CBDC is called DC/EP (digital currency and electronic payment), which has been studied and developed in the past five years, and is "forthcoming any time". Therefore, with the market of cryptocurrencies evolving so fast in recent years, it has prompted central banks all over the world to assess the possibility of issuing CBDC, including but not limited to the Federal Reserve Bank of U.S., Bank of Canada, PBC of China, Sveriges Riksbank (Sweden), Swiss National Bank, and so on. Furthermore with new technologies (blockchain, AI, big data, mobile payment, etc.) available, it is the time for central banks to re-examine the ways they issue and control fiat money.

Given the forthcoming CBDC, our paper addresses these research questions: firstly, how should we design CBDC? One advantage of CBDC is that it can pay interests. Then, once introducing interest-bearing CBDC, how would it affect financial intermediation, and then monetary policy conducting? Can the CBDC interest rate become a new policy tool? Or even go negative to conduct the negative-interest-rate (NIR) policy, as what has happened in Japan, Euro Zone and some European countries in recent years? In the end, what are the impacts on investment and real economy?

Before answering these questions, we need clearly define CBDC. First, CBDC is *fiat digital money*, *not* private cryptocurrency per se. Fundalmentally, CBDC is "centralized", since it is central bank high power money, directly issued and controlled by a central bank. This makes it very different from private cryptocurrencies which are "decentralized", and can support peer-to-peer settlements, either issued through some algorithm (like Bitcoin), or by some private enterprises (like Libra).

Second, CBDC is also different from cash. Although both of them belong to fiat money, obviously, they are in different forms: CBDC are in digital forms while cash is physical paper money. For example, individuals or firms may open CBDC accounts through an independent CBDC infrastructure, or through the current banking/settlement infrastructure. Nowadays cash is used less and less for transactions, in advanced economies such as Canada, U.S., Switzerland, Denmark, UK, etc. (Engert et al. 2019), and also in emerging economies like China. On one hand, this is due to the shortcomings of cash, e.g., costly to produce (the cost of printing, counterfeiting technology development, etc.), costly to carry, and hard to track the transactions (used in anonymous transcations, so cash like U.S. dollars circulates in underground economy). On the other hand, it is also due to the technology progress, e.g., widely usage of credit cards and bank cards in advanced economies, and quickly catching up in Fintech (mobile payment, big data, AI., etc.) in emerging economics like China and some African countries. Furthermore, it is possible to pay interests for CBDC. This is another stark difference between CBDC and paper money, since it is impossible to pay interests to cash. From a policy perspective, CBDC interest rate can become a new policy tool, and central bank can set the interest rate as positive or negative, when necessary.

Third, we can also compare it to bank deposits, since both of them are in the form of "digital accounts", and seem very close to each other. Indeed, bank deposits are "inside money" while CBDC belongs to "outside money". However, the key difference is that there is no insolvency issues for CBDC, as bank deposits may have these issues from financial institutions.[1] As shown in the left panel of Figure 1, the trend of M0/GDP ratio for selected countries during 1945 – 2018. It displays a U shape, i.e., decreasing firstly then increasing again in 2008, particularly for Switzerland and U.S. The former is due to financial innovations in 1980-1990s, when debit and credit cards replaced cash payment a lot, while the latter showed the rapid increasing demand for cash after the 2008 Global Financial Crisis (and the subsequent Euro Crisis) broke out. Cash is perceived as an insurance device against the insolvency of financial institutions in the crisis. For example, the right panel of Figure 1 shows the large and small denomimations as a ratio of GDP in Canada. It clearly shows the increasing demand for large denominations justifies the slightly upward trend of cash/GDP ratio in Canada after 2008. Obviously, there is great demand for a virtual asset issued by a trusted party, e.g., CBDC, that can be used to save outside of the private financial

[1] Here we suppose, in general, central bank of a country is trustworthy, and can function well as the last resort for the whole financial system, while commercial banks may have insolvency issues, particularly in the time of crisis. Obviously, we exclude the extreme cases of untrustworthy central banks as in Latin American and other countries.

system, particularly during the crisis time.

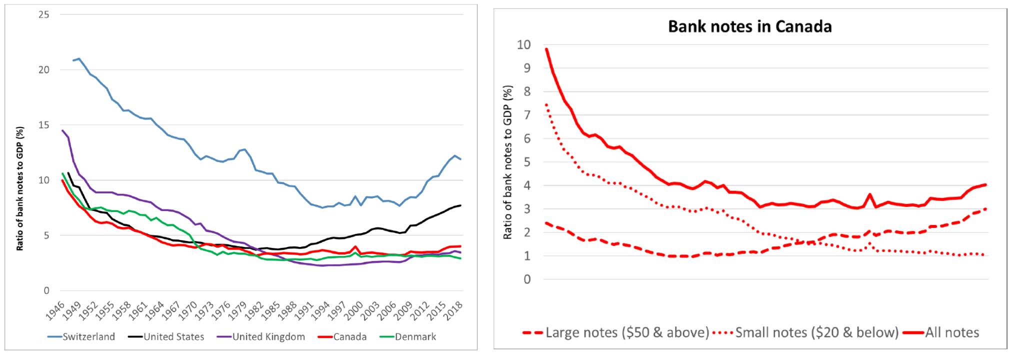

Figure 1: M0/GDP Ratio for Selected Countries (left); Ratios of Large&Small Denominations for Canada (right).

Source: Engert et al. (2019), Chart 3a (1945-2018), 5a (1946-2018)

After clearly defining CBDC, we now explain what we do in this paper, to tackle the above research questions. We start from a benchmark model with only CBDC, where we explicitly model a frictional deposit market and a frictional loan market. Entrepreneurs hold CBDC, and may or may not have investment opportunities. If they do not have investment opportunities (labeled as type-0 entrepreneurs), they deposit the idle CBDC at banks in a frictional deposit market. If they do ((labeled as type-1 entrepreneurs), they use CBDC as down payment, then apply for bank loans in a frictional loan market, to acquire capital and produce final output.

CBDC is interest bearing, so there are two monetary policy tools available: one is a traditional tool of changing money growth rate (equivalent of changing inflation rate at steady states), and the other is a new tool of changing CBDC interest rate. We label this model as "real CBDC" version, since we use real balances of CBDC, real deposit rate and other real variables for notations. In the "real" version, banks can access to CBDC. That is, if banks hold CBDC, they will also be paid the interests. Then, we consider another version of "nominal CBDC", where banks cannot access to CBDC, and most key variables are in nominal terms (see Appendix A). We examine these two versions because it is a key design issue whether banks have access to CBDC and get paid interests or not. Our analytical results do support this point very well, since changing CBDC interest rate causes different policy effects in these two versions.

An alternative way to interpret the benchmark model is that it resembles the case where CBDC totally phases out paper money, and becomes a universal medium of exchange for the economy. It is also an ideal scenario for us to think about the effects of conducting NIR policy. In the "real" version, the Euler equation for CBDC shows the cost of holding CBDC, in fact, is a "spread" which captures the interest rate difference between an illiquid bond and CBDC. Hence, there is room for the central bank to set the CBDC interest rate to a negative level.

Our main result from the benchmark model is that a higher CBDC interest rate tends to have a positive impact on investment. This is in sharp contrast with findings in existing models of CBDC. For example, in Andolfatto (2018) and Keister and Sanches (2018), CBDC and bank deposits serve as substitutes so that a higher CBDC interest rate tends to crowd out deposits and reduce investment. An exception is in Chiu et al. (2019) where CBDC and bank deposits are still substitute. Owing to the imperfect competition in the deposit market, a higher CBDC interest rate may help limit bank's market power and force banks to offer a higher deposit rate to prevent people from switching bank deposits to CBDC. Therefore, deposits and loans increase in response to the higher CBDC interest rate. In their model, the CBDC interest rate serves as a floor for the deposit rate. The critical difference between our model and these existing models is that CBDC and banks are complements in the spirit of Berentsen et al. (2007). Banks help channel liquidity from entrepreneurs who have idle CBDC to those who need more CBDC. The complementarity between CBDC and bank deposits makes a higher CBDC interest rate more favorable to deposits and investment. The important message from the benchmark model is that the banking structure matters when it comes to assessing the macroeconomic effects of CBDC.

To consider more CBDC design issues, we then extend the benchmark model by adding cash to the portfolio of entrepreneurs.[2] Suppose banks cannot access to CBDC,

[2] In the benchmark model or the extended one, we model entrepreneurs hold CBDC, or the portfolio of cash and CBDC. Someone may not feel it as intuitive as modelling individuals holding CBDC or the portfolio for daily transactions. However, corporate cash holding has been an important issue for firms in the U.S. and other advanced economies since 1980s (see Bates et al. 2009, Azar et

instead can help store CBDC, but only accept cash as deposits. The main results from this extended model include two dimensions: firstly, cash and CBDC can coexist only when banks' reserve constraint binds (when it does not, coexisting requires the CBDC interest rate to be zero); secondly, when cash and CBDC coexist, there is a redistribution effect when changing the CBDC interest rate, which makes unbanked entrepreneurs increase investment, and banked entrepreneurs reduce investment. To consider alternative designs of CBDC, we also consider another version of extension in Appendix B, which features coexistence of cash and CBDC, banks can access to CBDC, and accept both cash and CBDC as deposits. This is based on the "nominal CBDC" benchmark model. The main results show cash and CBDC can coexist only when the CBDC interest rate is zero (same as cash).

Adding cash to the benchmark is mainly to capture the initial stage of issuing CBDC, since it is more realistic that cash and CBDC coexists. Although no country has issued CBDC yet (China or Sweden may be the first country to do so), a good reference is to see what happened in the banknote demonetisation of India. On 8 November, 2016, the Government of India announced the demonetisation of all NIR 500 and 1000 banknotes of the Mahatma Gandhi Series, over a period of fifty days until 30 December, 2016 (Wikipedia). It also announced the issuance of new NIR 500 and 2000 banknotes in exchange for the demonetized ones. Hence, the demonetisation can be regarded as a "natural experiment" of changing fiat money system. Indeed, what happened later in India proves that it is normal to have the "old" money and "new" money coexist during some *short* period for transition. In the extended models with CBDC and cash, one main message is that coexisting of CBDC and cash is very tricky, as in the real world, the coexisting period should be temporary. Since CBDC is interest-bearing, easily one "money" will crowd out another, e.g., with positive interest rate of CBDC, no one is willing to hold cash, while with negative interest rate of CBDC, no one is willing to hold CBDC. Hence, to consider NIR policy, it is

al. 2015, Graham and Leary 2018 and many other corporate finance papers). Graham and Leary (2018) documented the current level of average cash holdings is around 25% of assets, for US firms. Furthermore, this issue is highly related to the financing decision of firms, as we explicitly show in the paper, i.e., the internal and external finance issues of firms.

more feasible to conduct in an economy with a universal CBDC.

To sum up, we build models with interest-bearing CBDC and explicit modelling of a frictional deposit market and a frictional loan market, to explore various designs of CBDC, then study the effects of issuing CBDC on banking and macroeconomy. We address a frontier and policy-oriented research topic, and also contribute to the monetary theory literature by providing a framework to analyze how introducing CBDC affects banking in both the liability-side (deposit) and asset-side (loan) operations, and affects monetary policy conducting by introducing a new policy tool, i.e., CBDC interest rate. We also address other important design issues of CBDC such as accessibility of CBDC to banks.

Literature Review Our paper is related to three lines of literature. The first line is literature related to CBDC, including Keister and Sanches (2018), Andolfatto (2018), and Chiu et al. (2019). There are also a few policy reports on CBDC, such as Bordo and Levin (2017) and Berentsen and Schar (2018). This literature has not had many papers since CBDC belongs to very new and frontier research.

Keister and Sanches (2018) build a model where both central bank money and private bank deposits are used in exchange, to study the effects of introducing CBDC on interest rates, economic activity, and welfare. They have competitive banking, and CBDC and bank deposits as substitutes in the model. Their results show that introducing CBDC tends to promote efficiency in exchange and raises welfare, but also crowds out bank deposits and decreases investment. In contrast, with the setting of non-competitive banking, Andolfatto (2018) and Chiu et al. (2019) both study the impacts of issuing CBDC on banking. Their difference is that Andolfatto (2018) uses an OLG model with monopolistic banking, while Chiu et al. (2019) use the framework of New Monetarism model with a competitive loan market, but a cournot-oligopolistic deposit market.

In all of these three papers, CBDC and bank deposits are modelled as substitutes in exchange, which is very different from our complementary setting of CBDC and bank deposits. In addition, the focus of our paper is different from these papers. They focus on the impacts of CBDC on banking, investment and welfare, while our paper not only gets involved with these aspects, but also focuses on how effective CBDC interest rate can be a new monetary policy tool, and discusses the policy of negative interest rate.[3] Our extended models with both CBDC and cash also address important CBDC design issues, while this coexisting issue is either ignored or not fully addressed in those papers. Furthermore, we clearly focus on a corporate finance perspective, which is very different from those papers as well.

The second line is banking literature. There are many papers on banking since the canonical paper of Diamond & Dyvbig (1983). Here we just list a few that are highly related to our paper. Banks in our models accept idle liquidity as bank deposits from those who do not need liquidity, and then make loans to those who need. This is also the key mechanism to make CBDC and bank deposits become complements in the models. The role of banks is similar to Berentsen et al. (2007). However, they focus on a consumer finance perspective in the model, where there is no capital, agents are consumers, and banking is perfectly competitive, both in the deposit market and the loan market. In contrast, we focus on a corporate finance perspective, where agents are entrepreneurs, and we model both the deposit market and loan market as frictional ones. Our paper is also related to Rocheteau et al. (2018b) in the corporate finance perspective. The frictional loan market is similar to theirs, but we also explicitly model a frictional deposit market, which is absent in their paper. Not withstanding the totally different focuses: we focus on CBDC and the effects of introducing CBDC on banking and macroeconomy, while they focus on the pass-though and transmission mechanism of monetary policy from a corporate finance perspective. There are also a lot of other papers studying banking, such as Williamson (2012), Gu et al. (2013), Brunnermeier and Sannikov (2016), Dong et al. (2017), etc.

The third line of literature is about cryptocurrency and blockchain, including Chiu and Koeppl (2017), Hendry and Zhu (2017), Huberman et al. (2017), Abadi

[3]There are some papers related to negative interest rates, including He et al. (2008), Rocheteau et al. (2018a), Dong and Wen (2017), and Groot and Haas (2018). He et al.(2008) and Rocheteau et al. (2018a) use New Monetarism models and can generate negative interest rate for assets. Dong and Wen (2017) and Groot and Haas (2018) study the negative interest rate policy which has happened in some advanced economies (such as Japan, Euro Zone, and some European countries), but neither of them is related to CBDC.

and Brunnermeier (2018), Schilling and Uhlig (2018), Dong et al. (2019), etc. These papers help understand cryptocurrency and blockchain technology, particularly how cryptocurrencies are different from fiat money. Our paper differs from these papers since CBDC is not cryptocurrency per se.

The rest of the paper is organized as follows. Section 2 describes the environment. Section 3 introduces the benchmark model, where CBDC is the only medium of exchange (when it totally phases out cash), and banks can access to CBDC. Section 4 extends the benchmark model by adding cash, to capture the initial stage of introducing CBDC when it coexists with cash, but banks can only accept cash as deposits. Section 5 discusses more design issues on CBDC, and concludes the paper. To consider various designs of CBDC, we also present a nominal version of the benchmark model in Appendix A, where banks cannot access to CBDC; and another version of the extended model in Appendix B, where CBDC and cash coexist, and banks can accept both as deposits.

2 Environment

Time is discrete and continues forever. Each period has three stages. Stage 1 is a decentralized deposit market. Stage 2 has a decentralized loan market, and a competitive capital market operating in parallel Stage 3 is a centralized market (CM). There are three types of agents: entrepreneurs (e), suppliers (s) and banks (b). There is a measure 1 of entrepreneurs, who are subject to an investment shock. With a probability n where $n > 1/2$, an entrepreneur has an investment opportunity and needs to acquire capital for production. With the rest probability $1-n$, the entrepreneur does not have an investment opportunity. We label them as type-1 and type-0 entrepreneurs, respectively. The investment shock is realized at the beginning of each period. Suppliers can provide capital in the capital market. As in Rocheteau et al. (2018b), the measure of suppliers is irrelevant due to constant returns. There is a measure 1 of banks that can take deposits in the deposit market can issue loans in the loan market. Banks are owned by all entrepreneurs equally.

In the benchmark model, we assume that the only asset available is CBDC. This scenario resembles the case when CBDC completely phase out paper money. We will consider the coexistence of money and CBDC in Section 4. CBDC is fiat digital money issued by central bank, with the price ρ, measured by CM numeriare goods x, and the nominal interest rate i_c paid per period. Let M denote the supply of CBDC by the central bank. There are two types of monetary policy tools. One is to change the growth rate of CBDC,

$$\frac{M}{M_{-}} = 1 + \mu,$$

where $1 + \mu \equiv \rho/\hat{\rho}$, and $1 + \mu = 1 + \pi$ at steady states (π is inflation rate). Based on the Fisher equation, $1+i = (1+\pi)/\beta$, changing π is equivalent of changing i at steady states. Here i can be interpreted as the nominal interest rate of illiquid bonds, which can measure the opportunity cost of holding fiat money. We assume that banks can take CBDC as deposits. For banks to issue loans, they need to take deposits firstly and satisfy a reserve requirement. The timeline of a representative period is shown in Figure 2, and the details of each stage are as follows.

In the first stage, all banks go to the deposit market to take deposits in order to make loans in the subsequent loan market. After the investment shock is realized, type-0 entrepreneurs go to the deposit market to deposit their idle balances. We assume a simple matching technology in the deposit market: short-side being served. That is, given the measure of type-0 entrepreneurs in the deposit market is $1 - n$, the probability of matching for entrepreneurs is 1 and the probability of matching for bankers is $1 - n$ in the deposit market. Those bankers who do not get deposits will not proceed to the loan market as there is a reserve requirement which requires bankers to hold a fraction υ of total assets in the form of reserves (CBDC).[4] Bankers and entrepreneurs bargain over the terms of the deposit contract. Notice that we use search and bargaining to capture the frictional deposit market. In the real world, entrepreneurs are relatively more important customers to banks, in terms of the deposit size and other business with banks, compared to individual customers. Hence, it is

[4]Notice that, to include the reserve requirement, Rocheteau et al.(2018b) introduce an interbank market where bankers can borrow at some policy rate, while we introduce a loan market to do so.

natural that they may "shop" around among various banks and bargain for better terms of deposit contracts.

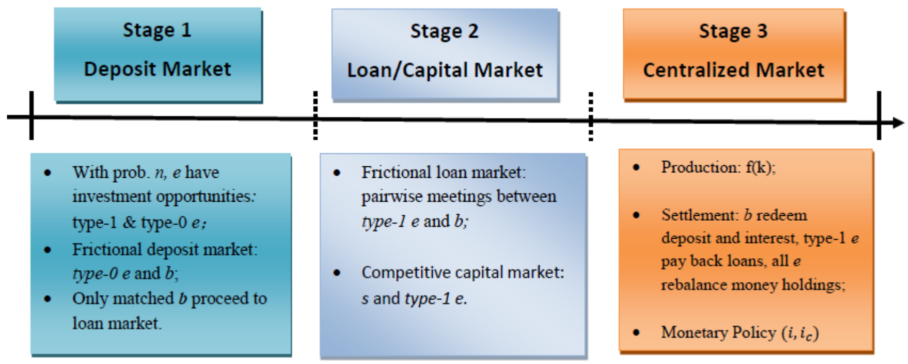

Figure 1: Timeline of a Representative Period

Those bankers who obtain deposits and all type-1 entrepreneurs participate in the second stage: the loan market. For simplicity, we again assume the simple matching technology: short-side being served. Given that the measure of entrepreneurs is n and the measure of bankers is $1-n$, the probability of matching for entrepreneurs is $(1-n)/n$ and the probability of matching for bankers is 1 because we assume $n > 1/2$. Bankers and entrepreneurs bargain over the terms of the loan contract, including a down payment p (in the form of CBDC), loan service fees ϕ and the loan size ℓ. Such a contract implies that the real loan rate is $r_\ell = \phi/\ell$. If an entrepreneur does not meet a banker, the entrepreneur uses internal finance to purchase capital in the competitive capital market, where suppliers provide capital at the market price q_k.

In the third stage, all agents can participate the competitive market. Entrepreneurs who deposited in the first stage redeem their deposits and entrepreneurs who borrowed in the loan market repay the loans and banking service fees. Bankers distribute all profits to entrepreneurs. Entrepreneurs use capital for production. As for the government, we suppose it is a consolidated monetary and fiscal authority, and only active in the third stage. The government can use both μ and i_c as monetary policy tools. We allow $i > i_c \geq 0$ or $i_c < 0$. When $i_c < 0$, it resembles the scenario that the central bank conducts NIR policy. The budget constraint of the government is

$$G + T = (\pi - i_c)M, \tag{1}$$

where G is government spending, T is lump-sum transfers. The LHS in (1) refers to the total government expenditure, while the RHS is the seigniorage revenues net of CBDC interest payment.

3 Benchmark Model

We begin from the third stage of the current period, then the first and second stage next period. In the beginning of the third stage, there are two types of entrepreneurs: type-1 entrepreneurs who have an investment opportunity in this period and type-2 entrepreneurs who do not have an investment opportunity. We use $W_1^e(z_c, \ell, k)$ to represent the value function of type-1 e, with a portfolio (z_c, ℓ) and capital holding k. Here $z_c \equiv \rho m_c(1 + i_c)$ denotes the real value of CBDC including the interest, held by an entrepreneur, and ℓ denotes the amount of loans if the entrepreneur borrows in the previous loan market. For type-1 e,

$$W_1^e(z_c, \ell, k) = \max_{x, \hat{z}_c} \{x + \beta \mathbb{E} U^e(\hat{z}_c)\}$$
$$\text{st. } x + \frac{\hat{z}_c}{1 + r_c} = z_c - \ell + f(k) + T + \Pi,$$

where T and Π represent transfers from the government and profits distributed by banks. The real interest rates of CBDC can be calculated by

$$1 + r_c = \frac{1 + i_c}{1 + \mu}, \tag{2}$$

where i_c is the nominal interest rate paid on CBDC chosen by the central bank and μ is the inflation rate. The unconstrained maximization problem is

$$W_1^e(z_c, \ell, k) = z_c - \ell + f(k) + T + \Pi + \max_{\hat{z}_c} \left\{ -\frac{\hat{z}_c}{1 + r_c} + \beta \mathbb{E} U^e(\hat{z}_c) \right\}.$$

We use $W_0^e(z_c, 0, 0)$ to represent the value function of type-0 e. They only hold CBDC, and do not hold any capital, or need to pay back bank loans. For type-0 e,

$$W_0^e(z_c, 0, 0) = \max_{x, \hat{z}_c} \{x + \beta \mathbb{E} U^e(\hat{z}_c)\} \text{ st. } x = z_c + T + \Pi - \frac{\hat{z}_c}{1 + r_c}.$$

The unconstrained maximization problem is

$$W_0^e(z_c, 0, 0) = z_c + T + \Pi + \max_{x, \hat{z}_c} \left\{ -\frac{\hat{z}_c}{1 + r_c} + \beta \mathbb{E} U^e(\hat{z}_c) \right\}.$$

It is clear that entrepreneurs will choose the same $\hat{z}_c$ independent of their previous status

$$\frac{1}{1 + r_c} = \frac{\beta \partial \mathbb{E} U^e(\hat{z}_c)}{\partial \hat{z}_c}. \tag{3}$$

As for bankers, the value function is

$$W^b(\omega) = \max_x \{x + \beta U^b\} \text{ st. } x = \omega + T,$$

where ω refers to the wealth bankers have at the third stage. Hence,

$$W^b(\omega) = \omega + T + \beta U^b,$$

which shows the value function of bankers is linear in ω. There is a similar value function for suppliers.

Moving to the first stage in the next period, the investment shock is realized. We use $U_1^e(\hat{z}_c)$ to denote the value function of entrepreneurs who have an investment opportunity, where $U_1^e(\hat{z}_c) = V_1^e(\hat{z}_c)$ .These entrepreneurs do not deposit in the deposit market because they will not be able to withdraw their deposits in the loan market to purchase capital. For type-0 e,

$$U_0^e(\hat{z}_c) = V_0^e[\hat{z}_c - d + (1 + r_d) d],$$

where (d, r_d) represent the terms of the deposit contract. The entrepreneur negotiates with the banker on the amount to deposit d and the real interest rate paid on deposits. We require $d \leq \hat{z}_c$. Notice that

$$\mathbb{E}U^e(\hat{z}_c) = nU_1^e(\hat{z}_c) + (1-n)U_0^e(\hat{z}_c).$$

In the second stage, type-0 entrepreneurs do not participate the loan market. Only type-1 entrepreneurs and bankers who attract deposits participate the loan market. Let $V_0^e(\hat{z}_c + r_d d)$ and $V_1^e(\hat{z}_c)$ denote their value functions, then

$$\begin{aligned} V_0^e(\hat{z}_c + r_d d) &= W_0^e(\hat{z}_c + r_d d, 0, 0) \\ V_1^e(\hat{z}_c) &= \frac{1-n}{n} W_1^e(\hat{z}_c - p_b, q_k k_b - p_b + \phi, k_b) \\ &+ (1 - \frac{1-n}{n}) W_1^e(\hat{z}_c - p_z, 0, k_z), \end{aligned}$$

where $(1-n)/n$ is the matching probability for a type-1 entrepreneur. We use subscript b to denote terms associated with banked type-1 entrepreneurs, i.e., they get bank loans, and subscript z to denote terms associated with unbanked type-1 entrepreneurs. For example, p_b and p_z refer to the amounts of payment to purchase capital k_j through internal finance by banked entrepreneurs and unbanked entrepreneurs, respectively.

For bankers, if they successfully obtain deposits in stage 1, their value function is

$$U^b = (1-n)V^b\left[d - (1 + r_d)\, d\right],$$

where d represents the amount of deposits available to be used as reserves and $(1 + r_d)\, d$ represents the promised repayment to the depositor. In the second stage, the banker's value function is $V^b(-r_d d) = W^b(\phi - r_d d)$ .In the third stage, the bank's profit is given by $W^b(\phi - r_d d) = \phi - r_d d$,which is fully distributed to entrepreneurs.

For suppliers at the capital market,

$$V^s = \max_k \{-k + W^s(q_k k)\},$$

which leads to $q_k = 1$.

3.1 Bargaining

We assume that entrepreneurs and bankers bargain following the Nash bargaining protocol in both the deposit market and the loan market. In the deposit market, let γ be the bargaining share of entrepreneurs. If a deposit contract is agreed upon, the surplus for the entrepreneur is $(1 + r_d)\, d - d = r_d d$ and the surplus for the bank is $V^b\,(-r_d d)$. The Nash bargaining problem is

$$\max_{d, r_d} (r_d d)^\gamma \left[V^b\,(-r_d d)\right]^{1-\gamma} \text{ st. } d \leq \hat{z}_c.$$

In the loan market, a bank and a type-1 entrepreneur bargain over the terms of contract (p_b, k_b, ϕ), where p_b is the downpayment by CBDC and ϕ is the banking service fee. Suppose the bargaining share of banks is θ. By Nash bargaining, we have

$$\max_{k_b, p_b, \phi} \left[f\,(k_b) - k_b - \phi - \Delta_z\,(\hat{z}_c)\right]^{1-\theta} \phi^\theta$$

$$\text{st. } k_b - p_b + \phi \leq \chi f\,(k_b) \tag{4}$$

$$k_b - p_b \leq d\left(\frac{1}{\upsilon} - 1\right) \tag{5}$$

$$p_b \leq \hat{z}_c,$$

where

$$\Delta_z\,(\hat{z}_c) = f\,(k_z) - k_z.$$

The first inequality (4) indicates the collateral constraint for a type-1 entrepreneur: he uses χ fraction of the final output $f(k)$ as collateral, to get bank loans. The second inequality (5) indicates the reserve constraint for a bank: the amount of lending is constrained by the amount of reserves held by the bank. The reserve requirement enforces banks to hold a υ fraction of assets as reserves in the form of CBDC. For a bank with deposits d, the maximum amount of loans issued by the bank is $d\,(1/\upsilon - 1)$. We can define $\delta \equiv 1/\upsilon - 1$ as the loan to reserve ratio.

We first solve for the terms of trade in the loan market, taking $(d, \hat{z}_c)$ as given. This is because $\hat{z}_c$ is determined by entrepreneurs in the previous centralized market and d is determined by bankers in the previous deposit market. As for the loan market, we focus on the case $\hat{z}_c < k^*$. When $i_c < i$, banked entrepreneurs do not have incentives to hold more CBDC than the amount needed to finance k^*. It is intuitive to have $p_b = \hat{z}_c$ since entrepreneurs should weakly prefer to use all the CBDC as downpayment. Then the bargaining problem can be rewritten as

$$\max_{k_b,\phi} \left[f\left(k_b\right) - k_b - \phi - \Delta_z\left(\hat{z}_c\right)\right]^{1-\theta} \phi^{\theta}$$

$$\text{st. } k_b - \hat{z}_c + \phi \leq \chi f\left(k_b\right) \tag{6}$$

$$k_b - \hat{z}_c \leq \delta d. \tag{7}$$

We then set up the Lagrangian function,

$$\begin{aligned}\mathcal{L}(k_b, \phi, \lambda_1, \lambda_2) = & \left[f\left(k_b\right) - k_b - \phi - \Delta_z\left(\hat{z}_c\right)\right]^{1-\theta} \phi^{\theta} \\ & + \lambda_1\left[\chi f\left(k_b\right) - k_b + \hat{z}_c - \phi\right] \\ & + \lambda_2\left(\delta d - k_b + \hat{z}_c\right).\end{aligned}$$

The FOCs against k_b and ϕ are,

$$\frac{(1-\theta)\,\phi^{\theta}\left[f'\left(k_b\right) - 1\right]}{\left[f\left(k_b\right) - k_b - \phi - \Delta_z\left(\hat{z}_c\right)\right]^{\theta}} = \lambda_1\left[1 - \chi f'\left(k_b\right)\right] + \lambda_2 \tag{8}$$

$$\frac{\phi^{\theta-1}\{\theta[[f\left(k_b\right) - k_b - \phi - \Delta_z\left(\hat{z}_c\right)] - (1-\theta)\,\phi\}}{\left[f\left(k_b\right) - k_b - \phi - \Delta_z\left(\hat{z}_c\right)\right]^{\theta}} = \lambda_1. \tag{9}$$

Depending on whether the collateral constraint (6) and the reserve constraint (7) are binding, we have four cases to consider to solve for the loan contract.

Case 1: $\lambda_1 = 0$, $\lambda_2 = 0$. Neither constraint is binding, and we have $k_b = k^*$ from (8). The banking fee is solved from (9)

$$\phi = \theta \left[f\left(k^*\right) - k^* - \Delta_z\left(\hat{z}_c\right) \right]. \tag{10}$$

When both entrepreneurs and banks are not constrained, entrepreneurs can bargain to borrow to purchase the optimal amount of k.

Case 2: $\lambda_1 > 0$, $\lambda_2 = 0$. The collateral constraint binds, but the reserve constraint does not bind. We can use (8), (9) and the collateral constraint to solve for (k_b, ϕ, λ_1). In particular, (k_b, ϕ) satisfy

$$\frac{\theta \left[1 - \chi f'\left(k_b\right)\right]}{(1-\theta)(1-\chi) f'\left(k_b\right)} = \frac{\chi f\left(k_b\right) - k_b + \hat{z}_c}{(1-\chi) f\left(k_b\right) - \hat{z}_c - \Delta_z\left(\hat{z}_c\right)}, \tag{11}$$

$$\phi = \chi f\left(k_b\right) - k_b + \hat{z}_c. \tag{12}$$

Either (8) or (9) gives λ_1.

Case 3: $\lambda_1 = 0$, $\lambda_2 > 0$. The collateral constraint does not bind, but the reserve constraint binds. We solve for k from (7)

$$k_b = \delta d + \hat{z}_c. \tag{13}$$

The solution of ϕ can be found from (9)

$$\phi = \theta \left[f\left(k_b\right) - k_b - \Delta_z\left(\hat{z}_c\right) \right] \tag{14}$$

and (8) gives λ_2.

Case 4: $\lambda_1 > 0$, $\lambda_2 > 0$. Both the collateral constraint and the reserve constraint bind. We have k from (13) and ϕ from (12). The solution of (λ_1, λ_2) is found by solving (8) or (9).

Now we move back to the deposit market, to solve for the deposit contract. Since $V^b(-r_d d) = \phi - r_d d$, the bargaining problem can be rewritten as

$$\max_{d,r_d} (r_d d)^\gamma (\phi - r_d d)^{1-\gamma} \text{ st. } d \leq \hat{z}_c.$$

We consider two cases, depending on whether the deposit constraint $d \leq \hat{z}_c$ binds or not.

Case (a): $d \leq \hat{z}_c$ binds

If $d = \hat{z}_c$, we can solve for r_d from the FOC with respect to r_d

$$\gamma d [\phi - r_d d] = (1-\gamma)(r_d d)\left(d - \frac{\partial \phi}{\partial r_d}\right). \tag{15}$$

Notice that r_d does not affect ϕ from the solutions in the loan market, for Case 1–4. It follows that

$$r_d d = \gamma \phi, \tag{16}$$

where $d = \hat{z}_c$.

Case (b): $d \leq \hat{z}_c$ does not bind

If $d < \hat{z}_c$, the FOC for r_d is the same as (15) since we always have $\partial \phi / \partial r_d = 0$, then delivers $r_d d = \gamma \phi$. And the FOC for d gives,

$$\gamma(\phi - r_d d) + (1-\gamma) d \left(\frac{\partial \phi}{\partial d} - r_d\right) = 0. \tag{17}$$

From the loan market solution, d does not affect ϕ in Case 1 and 2, then the above (17) delivers the same result as in (16), except $d < \hat{z}_c$. Therefore, r_d and d are indeterminate in these two cases.

In Case 3 and 4, d does affect ϕ. In case 3,

$$\frac{\partial \phi}{\partial d} = \theta [f'(k_b) - 1] \frac{\partial k_b}{\partial d} = \theta \delta [f'(k_b) - 1] > 0,$$

and one can check the LHS of (17) becomes positive, since $r_d d = \gamma \phi$. It implies that the optimal choice is to have $d \to \hat{z}_c$ because a higher d can increases ϕ from the loan market. In case 4,

$$\frac{\partial \phi}{\partial d} = \left[\chi f'(k_b) - 1\right] \frac{\partial k_b}{\partial d} = \delta \left[\chi f'(k_b) - 1\right].$$

We can rule out $\chi f'(k_b) < 1$, because it would lead to $\partial\phi/\partial d < 0$, meaning more deposits lead to less banking service fees, then $d \to 0$. Then banks would not "survive" and make loans in Stage 2. Therefore, we should have $\partial\phi/\partial d = \delta\left[\chi f'(k_b) - 1\right] \geq 0$, and the LHS of (17) becomes positive or zero. When $\chi f'(k_b) > 1$, we have $\partial\phi/\partial d > 0$, and will get the same results as in Case 3, i.e., $d \to \hat{z}_c$, and the same bargaining solutions as when $d \leq \hat{z}_c$ binds. Only when $\chi f'(k_b) = 1$, there is an interior solution for d.

To sum up the results from Case (a) and (b), it might be without loss of generality to set $d = \hat{z}_c$, and r_d satisfies (16).

3.2 General Equilibrium

After solving for the deposit contract and the loan contract, we can use these conditions to find the choice of $\hat{z}_c$ in the centralized market. The term that is important for the determination of $\hat{z}_c$ is $\beta \mathbb{E} U^e(\hat{z}_c)$, where

$$\begin{aligned}
\mathbb{E}U^e(\hat{z}_c) &= nU_1^e(\hat{z}_c) + (1-n)U_0^e(\hat{z}_c) \\
&= (1-n)\Delta_b(\hat{z}_c) + (2n-1)\Delta_z(\hat{z}_c) + (1-n)r_d d + \hat{z}_c \\
&\quad + nW_1^e(0,0,0) + (1-n)W_0^e(0,0,0).
\end{aligned}$$

We use $\Delta_z(\hat{z}_c)$ as previously defined and $\Delta_b(\hat{z}_c) = f(k_b) - k_b - \phi$ to represent the surpluses for unbanked and banked type-1 entrepreneurs, respectively. For an unbanked type-1 entrepreneur, he uses internal finances and $k_z = \hat{z}_c$.Then

$$\frac{\partial \Delta_z(\hat{z}_c)}{\partial \hat{z}_c} = f'(k_z) - 1.$$

In the case of $\partial\Delta_b(\hat{z}_c)/\partial\hat{z}_c$, we should consider four cases from the solution of the loan contract. The surplus for a type-0 entrepreneur who deposits in the deposit market is $r_d d$. Given (16), $\hat{z}_c$ can affect $r_d d$ through ϕ. Now we should check how the bank's reserve may affect ϕ, depending on which type of solution we have in the loan market.

Case 1: $\lambda_1 = 0$, $\lambda_2 = 0$

Using $k_b = k^*$ and (10), we have,

$$\frac{\partial\Delta_b(\hat{z}_c)}{\partial\hat{z}_c} = -\frac{\partial\phi}{\partial\hat{z}_c} = \theta\frac{\partial\Delta_z(\hat{z}_c)}{\partial\hat{z}_c}.$$

Notice that ϕ is not affected by the bank's reserve. So $\partial(r_d d)/\partial\hat{z}_c = 0$ and (3) becomes

$$s = A\left[f'(k_z) - 1\right],$$

where $s \equiv (i - i_c)/(1 + i_c)$, $A \equiv (2n - 1) + (1 - n)\theta > 0$. Here s is the spread between i and i_c, measuring the marginal cost of holding CBDC, and $\partial s/\partial i_c = -(1+i)/(1+i_c)^2 < 0$. Obviously, it is optional for the central bank to set $i_c < 0$, since it is the "spread" s matters for the marginal cost to hold one more unit of CBDC. And it is straightforward that $\partial k_z/\partial i_c > 0$ given that $f''(k_z) < 0$. In terms of ϕ, $\partial\Delta_z(z_c)/\partial i_c > 0$ and $\partial\phi/\partial i_c < 0$. The deposit rate is

$$r_d = \frac{\gamma\phi}{z_c} = \frac{\gamma\phi}{k_z}. \tag{18}$$

It follows that

$$\frac{\partial r_d}{\partial i_c} = \frac{\gamma k_z\frac{\partial\phi}{\partial i_c} - \gamma\phi\frac{\partial k_z}{\partial i_c}}{k_z^2} < 0.$$

The loan rate is

$$r_\ell = \frac{\phi}{k^* - k_z},$$

and

$$\frac{\partial r_\ell}{\partial i_c} = \frac{(k^* - k_z)\frac{\partial\phi}{\partial i_c} + \phi\frac{\partial k_z}{\partial i_c}}{(k^* - k_z)^2} < 0$$

due to concavity of the production function.

In this case, entrepreneurs hold more CBDC when i_c increases. Since banked entrepreneurs borrow $k^* - k_z$ from banks, a higher i_c reduces the amount of bank lending and ϕ charged by banks. The bargaining solution from the deposit market indicates that the real deposit rate depends on the benefit of accepting deposits ϕ and the amount of deposits k_z. Given that k_z increases and ϕ increases in response to a rise in i_c, the real deposit rate decreases. The real lending rate r_ℓ depends on the benefit of lending ϕ and the amount of lending $k^* - k_z$. The increase in i_c makes ϕ smaller, but $k^* - k_z$ also goes down. Overall, the decrease in ϕ dominates the effect of i_c on the real lending rate r_ℓ.

Case 2: $\lambda_1 > 0$, $\lambda_2 = 0$

The solution of (k_b, ϕ) is given by (11) and (12). In addition, we have

$$\frac{\partial \Delta_b\left(\hat{z}_c\right)}{\partial \hat{z}_c} = \left[f'\left(k_b\right) - 1\right] \frac{\partial k_b}{\partial \hat{z}_c} - \frac{\partial \phi}{\partial \hat{z}_c}.$$

Since the deposit constraint does not bind, an entrepreneur's money holding does not affect ϕ earned by the bank that takes deposits from this entrepreneur. So $\partial\left(r_d d\right)/\partial \hat{z}_c = 0$ and (3) becomes

$$s = (2n-1)\left[f'\left(k_z\right) - 1\right] + (1-n)\left[(1-\chi) f'(k_b)\Omega(k_b, k_z) - 1\right],$$

where $dk_b/d\hat{z}_c \equiv \Omega(k_b, k_z) > 0$, with

$$\Omega(k_b, k_z) = \frac{(1-\theta)(1-\chi)f'(k_b) + \theta[1-\chi f'(k_b)]f'(k_z)}{(1-\chi)f'(k_b)[1-\chi f'(k_b)] - f''(k_b)\{\theta\chi[(1-\chi)f(k_b) - f(k_z)] + (1-\theta)(1-\chi)\phi\}},$$

and $\phi = \chi f(k_b) - k_b + \hat{z}_c$ for this case.

In this case, it is less clear to derive the effects of i_c analytically.

Case 3: $\lambda_1 = 0$, $\lambda_2 > 0$

The solution of (k_b, ϕ) is (13) and (14). In the deposit market, if a type-0 entrepreneur deposits, the amount of deposits can affect the banking fee. That is,

$$\frac{\partial (r_d d)}{\partial d} = \gamma \frac{\partial \phi}{\partial d} = \gamma\theta\delta \left[f'(k_b) - 1\right].$$

The solution of $\hat{z}_c$ from (3) is

$$s = A\left[f'(k_z) - 1\right] + (1-n)(1-\theta+\gamma\theta\delta)\left[f'(k_b) - 1\right].$$

Notice that from (13), k_b and k_z satisfy $k_b = (1+\delta) k_z$.We find that $\partial k_z / \partial i_c > 0$ and $\partial k_b / \partial i_c > 0$. In terms of ϕ,

$$\frac{\partial \phi}{\partial i_c} = \theta \left\{ \left[f'(k_b) - 1\right] \frac{\partial k_b}{\partial k_z} - \left[f'(k_z) - 1\right] \right\} \frac{\partial k_z}{\partial i_c}$$

An increase in i_c might lower or raise ϕ. The deposit rate in the deposit market is (18) and the loan rate is

$$r_\ell = \frac{\phi}{k_b - k_z} = \frac{\phi}{\delta k_z}. \tag{19}$$

It follows that the sign of $\partial r_d / \partial i_c$ is the same as the sign of $\partial r_\ell / \partial i_c$. Notice that if $\partial \phi / \partial i_c < 0$, we have $\partial r_d / \partial i_c < 0$ and $\partial r_\ell / \partial i_c < 0$. If $\partial \phi / \partial i_c > 0$, the sign of $\partial r_d / \partial i_c$ and $\partial r_\ell / \partial i_c$ is ambiguous.

Case 4: $\lambda_1 > 0$, $\lambda_2 > 0$

The solution of (k_b, ϕ) is given by (12) and (13). In the deposit market, the marginal value of $\hat{z}_c$ depends on how the amount of deposits affect ϕ,

$$\frac{\partial (r_d d)}{\partial d} = \gamma \frac{\partial \phi}{\partial d} = \gamma\delta \left[\chi f'(k_b) - 1\right].$$

The solution of $\hat{z}_c$ from (3) is

$$s = (2n-1)\left[f'(k_z) - 1\right] + (1-n)\left\{\left[(1-\chi) f'(k_b) - 1\right] + \gamma\delta\left[\chi f'(k_b) - 1\right]\right\}.$$

Again, since (13) is binding, k_z and k_b satisfy $k_b = (1+\delta) k_z$.

In case 4, an increase in i_c makes the spread smaller, which requires k_z increasing to satisfy the FOC. That is, we have $\partial k_z/\partial i_c > 0$ and $\partial k_b/\partial i_c > 0$. Moreover, we have

$$\frac{\partial \phi}{\partial i_c} = \left\{ (1+\delta)\, \chi f'(k_b) - \delta \right\} \frac{\partial k_z}{\partial i_c}.$$

We can also check how the change in i_c affects the deposit contract and the loan contract. In the deposit market, the interest rate on deposit is (18). We can show that

$$\frac{\partial r_d}{\partial i_c} = \frac{k_z \gamma \frac{\partial \phi}{\partial k_z} - \gamma \phi}{k_z^2} \frac{dk_z}{di_c} < 0.$$

The loan rate is defined as in (19) and $\partial r_\ell / \partial k_z < 0$.

When the reserve constraint binds, k_b becomes proportional to k_z. An increase in i_c leads to a higher k_z. The reserve constraint implies that k_b also increases as the amount of lending that a bank can offer is constrained by the amount of reserve. From the binding collateral constraint, the increase in i_c has two opposite effects on ϕ. On one hand, a higher i_c makes k_b, which contributes to an increase in ϕ. On the other hand, the higher i_c raises k_z and the amount of lending increases. Due to the binding collateral constraint, ϕ decreases because a banked entrepreneur can only afford $\chi f(k_b)$ including the amount of lending and the banking fee ϕ. More lending results in a lower ϕ. Overall, it is not clear how ϕ changes in response to a change in i_c. However, the effects of i_c on the interest rates r_d and r_ℓ are not ambiguous. In particular, the rises in the amounts of deposits and lending given by k_z and δk_z respectively dominate the effects on r_d and r_ℓ. Both r_d and r_ℓ decrease. A higher i_c makes CBDC a more attractive asset for entrepreneurs. When entrepreneurs hold more CBDC, there are more deposits and more bank lending. While investments increase, both the real deposit rate and the real lending rate decrease.

4 Extension: Money and CBDC

Now we extend the benchmark model by adding money as an additional assets. Then we can consider how money and CBDC interact. The environment remains very similar to the benchmark model, except that entrepreneurs can hold a portfolio of CBDC and money. Suppose they have the same real price ρ, and the same growth rate μ, since both of them are fiat money issued by the government, only in different forms. We also assume that banks can take money as deposits at Stage 1, but can only help entrepreneurs store CBDC.[5] This way of modeling CBDC can be found in Andolfatto (2018), where banks can help individuals to store CBDC, but cannot use CBDC to issue loans.

4.1 Value functions

We first consider the value functions of entrepreneurs. The value functions of entrepreneurs at the beginning of the centralized market have an additional state variable z that represents an entrepreneur's holding of money in real terms. The value function of an entrepreneur with an investment opportunity in this period is

$$W_1^e(z, z_c, \ell, k) = \max_{x,\hat{z},\hat{z}_c} \{x + \beta \mathbb{E} U^e(\hat{z}, \hat{z}_c)\}$$
$$\text{st. } x + \frac{\hat{z}}{1+r_z} + \frac{\hat{z}_c}{1+r_c} = z + z_c - \ell + f(k) + T + \Pi,$$

The real interest rate of money is given by $1 + r_z = 1/(1+\mu)$.The unconstrained maximization problem is

$$W_1^e(k, z, z_c, \ell) = z + z_c - \ell + f(k) + T + \Pi + \max_{\hat{z},\hat{z}_c} \left\{ -\frac{\hat{z}}{1+r_z} - \frac{\hat{z}_c}{1+r_c} + \beta \mathbb{E} U^e(\hat{z}, \hat{z}_c) \right\}.$$

The value function of an entrepreneur without an investment opportunity in this

[5] We consider an alternative version in Appendix B, where banks can take both cash and CBDC as deposits.

period is

$$W_0^e(z, z_c, 0, 0) = \max_{x,\hat{z},\hat{z}_c} \{x + \beta \mathbb{E} U^e(\hat{z}, \hat{z}_c)\} \text{ st. } x = z + z_c + T + \Pi - \frac{\hat{z}}{1+r_z} - \frac{\hat{z}_c}{1+r_c}.$$

The unconstrained maximization problem is

$$W_0^e(z, z_c, 0, 0) = z + z_c + T + \Pi + \max_{x,\hat{z},\hat{z}_c} \left\{ -\frac{z}{1+r_z} - \frac{z_c}{1+r_c} + \beta \mathbb{E} U^e(\hat{z}, \hat{z}_c) \right\}.$$

Again, the choice of $\hat{z}_c$ is independent of an entrepreneur's previous status

$$\frac{1}{1+r_z} = \frac{\beta \partial \mathbb{E} U^e(\hat{z}, \hat{z}_c)}{\partial \hat{z}}, \tag{20}$$

$$\frac{1}{1+r_c} = \frac{\beta \partial \mathbb{E} U^e(\hat{z}, \hat{z}_c)}{\partial \hat{z}_c}. \tag{21}$$

We can write down the value functions for entrepreneurs in the following deposit market, where

$$U_1^e(\hat{z}, \hat{z}_c) = V_1^e(\hat{z}, \hat{z}_c),$$

$$U_0^e(\hat{z}, \hat{z}_c) = V_0^e[\hat{z} - d + (1 + r_d)\, d, \hat{z}_c].$$

Notice that entrepreneurs can deposit only money, but not CBDC in the deposit market. We require $d \leq \hat{z}$. We have

$$\mathbb{E} U^e(\hat{z}, \hat{z}_c) = n U_1^e(\hat{z}, \hat{z}_c) + (1-n) U_0^e(\hat{z}, \hat{z}_c).$$

In the loan market, let $V_1^e(\hat{z}, \hat{z}_c)$ and $V_0^e(\hat{z} + r_d d, \hat{z}_c)$ denote their value functions, where

$$V_1^e(\hat{z}, \hat{z}_c) = \mathbb{E} W_1^e(\hat{z} - p_z, \hat{z}_c - p_c, q_k k + \phi - p_z - p_c, k),$$

$$V_0^e(\hat{z} + r_d d, \hat{z}_c) = W_0^e(\hat{z} + r_d d, \hat{z}_c, 0, 0).$$

We use (p_z, p_c) to denote the amount of downpayment made by the entrepreneur using money and CBDC, respectively.

The value functions of bankers and suppliers remain the same as in the benchmark model.

4.2 Bargaining

The Nash bargaining in the deposit market is

$$\max_{d,r_d} (r_d d)^\gamma \left[V^b(-r_d d)\right]^{1-\gamma} \text{ st. } d \le \hat{z}.$$

Notice that deposits can only take the form of money. In the loan market, we modify the bargaining problem to incorporate the additional asset,

$$\max_{k_b,p_z,p_c,\phi} [f(k_b) - k_b - \phi - \Delta_z(\hat{z}, \hat{z}_c)]^{1-\theta} \phi^\theta$$

$$\text{st. } k_b - p_z - p_c + \phi \le \chi f(k_b) \tag{22}$$

$$k_b - p_z - p_c \le \delta d \tag{23}$$

$$p_z \le \hat{z} \text{ and } p_c \le \hat{z}_c.$$

We define $\Delta_z(\hat{z}, \hat{z}_c)$ as

$$\Delta_z(\hat{z}, \hat{z}_c) = f(k_z) - k_z.$$

Similar to the benchmark model, we first solve for the terms of trade in the loan market taking $(d, \hat{z}, \hat{z}_c)$ as given. This is because $(\hat{z}, \hat{z}_c)$ are determined by entrepreneurs in the previous centralized market and d is determined by bankers in the previous deposit market. We focus on the case where $\hat{z} + \hat{z}_c < k^*$, $p_z = \hat{z}$ and $p_c = \hat{z}_c$.

Then the bargaining problem can be rewritten as

$$\max_{k_b,\phi} \left[f(k_b) - k_b - \phi - \Delta_z(\hat{z}, \hat{z}_c)\right]^{1-\theta} \phi^{\theta}$$

$$\text{st. } k_b - \hat{z} - \hat{z}_c + \phi \leq \chi f(k_b)$$

$$k_b - \hat{z} - \hat{z}_c \leq \delta d.$$

We can set up the Lagrangian function as

$$\begin{aligned}\mathcal{L}(k_b, \phi, \lambda_1, \lambda_2) &= \left[f(k_b) - k_b - \phi - \Delta_z(\hat{z}, \hat{z}_c)\right]^{1-\theta} \phi^{\theta} \\ &+ \lambda_1 \left[\chi f(k_b) - k_b + \hat{z} + \hat{z}_c - \phi\right] \\ &+ \lambda_2 \left(\delta d - k_b + \hat{z} + \hat{z}_c\right).\end{aligned}$$

The FOCs against k_b and ϕ are,

$$\frac{(1-\theta)\phi^{\theta}\left[f'(k_b) - 1\right]}{\left[f(k_b) - k_b - \phi - \Delta_z(\hat{z}, \hat{z}_c)\right]^{\theta}} = \lambda_1 \left[1 - \chi f'(k_b)\right] + \lambda_2 \tag{24}$$

$$\frac{\phi^{\theta-1}\{\theta[[f(k_b) - k_b - \phi - \Delta_z(\hat{z}, \hat{z}_c)] - (1-\theta)\phi\}}{\left[f(k_b) - k_b - \phi - \Delta_z(\hat{z}, \hat{z}_c)\right]^{\theta}} = \lambda_1. \tag{25}$$

Then we follow similar steps as in the benchmark model to discuss four cases of solution, depending on whether the collateral constraint and the reserve constraint bind.

Case 1: $\lambda_1 = 0$, $\lambda_2 = 0$. Neither constraint is binding, and we have $k_b = k^*$.

$$\phi = \theta \left[f(k^*) - k^* - \Delta_z(\hat{z}, \hat{z}_c)\right].$$

Case 2: $\lambda_1 > 0$, $\lambda_2 = 0$. The collateral constraint binds, but the reserve constraint does not bind. We can use (24), (25), and the collateral constraint to solve for (k_b, ϕ, λ_1).

Case 3: $\lambda_1 = 0$, $\lambda_2 > 0$. The collateral constraint does not bind, but the reserve constraint binds. We solve for k_b from the reserve constraint

$$k_b = \delta d + \hat{z} + \hat{z}_c. \tag{26}$$

The solution of ϕ can be found from (25)

$$\phi = \theta \left[f\left(k_b\right) - k_b - \Delta_z\left(\hat{z}, \hat{z}_c\right) \right]$$

and (24) gives λ_2.

Case 4: $\lambda_1 > 0$, $\lambda_2 > 0$. Both the collateral constraint and the reserve constraint bind. We have k_b from the reserve constraint (26) and ϕ from the collateral constraint,

$$\phi = \chi f\left(k_b\right) - k_b + \hat{z} + \hat{z}_c. \tag{27}$$

For the deposit contract, we can solve for (d, r_d) following exactly the same way as in the benchmark model with $\hat{z}$ replacing $\hat{z}_c$.

4.3 General Equilibrium

In the centralized market, we use (20) and (21) to find solution for an entrepreneur's choice of $(\hat{z}, \hat{z})$. In particular,

$$\begin{aligned}
\mathbb{E}U^e\left(\hat{z}, \hat{z}_c\right) &= nU_1^e(\hat{z}, \hat{z}_c) + (1-n)U_0^e\left(\hat{z}, \hat{z}_c\right) \\
&= (1-n)\,\Delta_b\left(\hat{z}, \hat{z}_c\right) + (2n-1)\,\Delta_z\left(\hat{z}, \hat{z}_c\right) + (1-n)\,r_d d + \hat{z} + \hat{z}_c \\
&+ nW_1^e\left(0,0,0,0\right) + (1-n)\,W_0^e\left(0,0,0,0\right),
\end{aligned}$$

where

$$\Delta_b\left(\hat{z}, \hat{z}_c\right) = f\left(k_b\right) - k_b - \phi.$$

We use subscript b to denote variables associated with entrepreneurs who use loans issued by banks and subscript z to denote variables associated with entrepreneurs who use only their own money and CBDC.

If a type-1 entrepreneur does not meet a bank, he uses his own internal finances and

$$k_z = \hat{z} + \hat{z}_c.$$

Then

$$\frac{\partial \Delta_z (\hat{z}, \hat{z}_c)}{\partial \hat{z}} = \frac{\partial \Delta_z (\hat{z}, \hat{z}_c)}{\partial \hat{z}_c} = f'(k_z) - 1.$$

In the case of $\partial \Delta_b (\hat{z}, \hat{z}_c) / \partial \hat{z}$ and $\partial \Delta_b (\hat{z}, \hat{z}_c) / \partial \hat{z}_c$, we should consider four cases of loan contract solution.

Case 1: $\lambda_1 = 0$, $\lambda_2 = 0$

We have,

$$\frac{\partial \Delta_b (\hat{z}, \hat{z}_c)}{\partial \hat{z}} = \theta \frac{\partial \Delta_z (\hat{z}, \hat{z}_c)}{\partial \hat{z}}.$$

Notice that ϕ is not affected by the bank's reserve. So $\partial (r_d d) / \partial \hat{z} = 0$. We have

$$\frac{\partial \mathbb{E} U^e (\hat{z}, \hat{z}_c)}{\partial \hat{z}} = (1-n) \frac{\partial \Delta_b (\hat{z}, \hat{z}_c)}{\partial \hat{z}} + (2n-1) \frac{\partial \Delta_z (\hat{z}, \hat{z}_c)}{\partial \hat{z}} + 1,$$
$$\frac{\partial \mathbb{E} U^e (z, z_c)}{\partial z_c} = (1-n) \frac{\partial \Delta_b (\hat{z}, \hat{z}_c)}{\partial \hat{z}_c} + (2n-1) \frac{\partial \Delta_z (\hat{z}, \hat{z}_c)}{\partial \hat{z}_c} + 1.$$

Then we can rearrange (20) and (21) as

$$i = A \left[f'(k_z) - 1 \right],$$

$$s = A \left[f'(k_z) - 1 \right],$$

where $A = 2n - 1 + \theta(1-n) > 0$, $s \equiv (i - i_c)/(1 + i_c)$, as defined in Section 3.

If $i_c > 0$, both Euler equations cannot be satisfied at the same time. In particular, since CBDC offers a higher rate of return, entrepreneurs would like to hold CBDC as much as possible and hold money as little as possible. However, when no entrepreneur holds money, banks cannot make loans. The economy effectively functions without banks. If $i_c < 0$, money has a higher return and CBDC will be driven out of existence. Type-0 entrepreneurs deposit their money in the deposit market and banked type-1 entrepreneurs can borrow in the loan market. Only when $i_c = 0$, money and CBDC can coexist, but as i_c is fixed at a special value, there is no meaningful interaction between money and CBDC.

Case 2: $\lambda_1 > 0$, $\lambda_2 = 0$

The solution of (k_b, ϕ, λ_1) is given by

$$\frac{(1-\theta)\,\phi^{\theta}\,[f'(k_b)-1]}{[f(k_b)-k_b-\phi-\Delta_z(\hat{z},\hat{z}_c)]^{\theta}\,[\chi f'(k_b)-1]} = -\lambda_1,$$
$$\frac{\theta\,[f(k_b)-k_b-\phi-\Delta_z(\hat{z},\hat{z}_c)]^{1-\theta}}{\phi^{1-\theta}} - \frac{(1-\theta)\,\phi^{\theta}}{[f(k_b)-k_b-\phi-\Delta_z(\hat{z},\hat{z}_c)]^{\theta}} = \lambda_1,$$
$$k_b - \hat{z} - \hat{z}_c + \phi = \chi f(k_b)\,.$$

We have

$$\frac{\partial \Delta_b(z,z_c)}{\partial z} = [f'(k_b)-1]\frac{\partial k_b}{\partial z} - \frac{\partial \phi}{\partial z} + 1,$$
$$\frac{\partial \Delta_b(z,z_c)}{\partial z_c} = [f'(k_b)-1]\frac{\partial k_b}{\partial z_c} - \frac{\partial \phi}{\partial z_c} + 1.$$

Since the deposit constraint does not bind, an entrepreneur's money holding does not affect ϕ earned by the bank that takes deposits from this entrepreneur. So $\partial(r_d d)/\partial \hat{z} = 0$ and

$$\frac{\partial \mathbb{E}U^e(\hat{z},\hat{z}_c)}{\partial \hat{z}} = (1-n)\frac{\partial \Delta_b(\hat{z},\hat{z}_c)}{\partial \hat{z}} + (2n-1)\frac{\partial \Delta_z(\hat{z},\hat{z}_c)}{\partial \hat{z}} + 1,$$
$$\frac{\partial \mathbb{E}U^e(\hat{z},\hat{z}_c)}{\partial \hat{z}_c} = (1-n)\frac{\partial \Delta_b(\hat{z},\hat{z}_c)}{\partial \hat{z}_c} + (2n-1)\frac{\partial \Delta_z(\hat{z},\hat{z}_c)}{\partial \hat{z}_c} + 1.$$

The FOCs (20) and (21) become

$$i = (2n-1)\,[f'(k_z)-1] + (1-n)\,[(1-\chi)f'(k_b)\,\Omega(k_b,k_z)-1]$$
$$s = (2n-1)\,[f'(k_z)-1] + (1-n)\,[(1-\chi)f'(k_b)\,\Omega(k_b,k_z)-1],$$

where $\Omega(k_b,k_z)$ is defined the same as Section 3, except $\phi = \chi f(k_b) - k_b + \hat{z} + \hat{z}_c$, while in Section 3, $\phi = \chi f(k_b) - k_b + \hat{z}_c$.

Similar to the previous case, entrepreneurs prefer to hold CBDC and money is driven out of existence when $i_c > 0$. The economy functions without banks. When $i_c < 0$, entrepreneurs prefer to hold money and no one holds CBDC. Both the deposit market and the loan market are active and money serves as the only asset in the economy.

Case 3: $\lambda_1 = 0$, $\lambda_2 > 0$

The solution of (k_b, ϕ) implies

$$\begin{aligned} \frac{\partial k_b}{\partial \hat{z}} &= \frac{\partial k_b}{\partial \hat{z}_c} = 1, \\ \frac{\partial \phi}{\partial \hat{z}} &= \theta[f'(k_b) - 1] - \theta \frac{\partial \Delta_z(\hat{z}, \hat{z}_c)}{\partial \hat{z}}, \\ \frac{\partial \phi}{\partial \hat{z}_c} &= \theta[f'(k_b) - 1] - \theta \frac{\partial \Delta_z(\hat{z}, \hat{z}_c)}{\partial z_c}. \end{aligned}$$

Then

$$\begin{aligned} \frac{\partial \Delta_b(\hat{z}, \hat{z}_c)}{\partial \hat{z}} &= (1-\theta)[f'(k_b) - 1] + \theta \frac{\partial \Delta_z(\hat{z}, \hat{z}_c)}{\partial \hat{z}}, \\ \frac{\partial \Delta_b(\hat{z}, \hat{z}_c)}{\partial \hat{z}_c} &= (1-\theta)[f'(k_b) - 1] + \theta \frac{\partial \Delta_z(\hat{z}, \hat{z}_c)}{\partial \hat{z}_c}. \end{aligned}$$

Now in the deposit market, if an entrepreneur deposits at the bank, the amount of deposits can affect the fee the bank can earn later. Therefore,

$$\frac{\partial (r_d d)}{\partial d} = \gamma \frac{\partial \phi}{\partial d} = \gamma\theta\delta [f'(k_b) - 1].$$

Overall, (20) and (21) become

$$i = A[f'(k_z) - 1] + (1-n)[(1-\theta) + \gamma\theta\delta][f'(k_b) - 1] \tag{28}$$

$$s = A[f'(k_z) - 1] + (1-n)(1-\theta)[f'(k_b) - 1],$$

which give the solution of (k_b, k_z). Then we can find $(\hat{z}, \hat{z}_c)$ and derive

$$i - s = (1 - n)\,\gamma\theta\delta\left[f'(k_b) - 1\right]. \tag{29}$$

It is worth noting that money and CBDC can coexist in this case. The binding reserve constraint implies that the interests earned on deposits depend on the banking fee, which in turn depends on the size of deposits. Compared to CBDC, money has the additional value through affecting the return on deposits. Therefore, when $i_c > 0$, there is a tradeoff between money and CBDC. While CBDC offers a lower marginal cost, money has a higher marginal value. Entrepreneurs generally hold a portfolio of money and CBDC. The coexistence of money and CBDC allows us to investigate how money and CBDC interact.

When i_c increases, the LHS of (29) increases, which implies that k_b should decrease. From (28), a lower k_b leads to a higher k_z. Notice that $k_b - k_z = \delta\hat{z}$. We know that $\hat{z}$ should decrease and $\hat{z}_c$ would increase as a result of a higher i_c. To summarize, we have

$$\frac{\partial \hat{z}}{\partial i_c} < 0,\ \frac{\partial \hat{z}_c}{\partial i_c} > 0,\ \frac{\partial k_z}{\partial i_c} > 0 \text{ and } \frac{\partial k_b}{\partial i_c} < 0 \tag{30}$$

The higher interest rate on CBDC induces entrepreneurs to hold more CBDC, which reduces their need for money. Since only money serves as reserves, type-0 entrepreneurs deposit less money and banks issue less loans to banked type-1 entrepreneurs. In this sense, the higher return CBDC crowds out deposits, which reduces the amount of lending in the economy. For type-1 entrepreneurs, unbanked entrepreneurs purchase capital using their own portfolios consisting of money and CBDC. It turns out that the increase in the holding of CBDC dominates the decrease in the holding of money. Unbanked entrepreneurs are able to raise k_z in response to a higher i_c. In contrast, despite that banked entrepreneurs' own portfolios allow them to purchase more capital, the reduction in bank lending leads to a lower k_b in response to a higher i_c.

Case 4: $\lambda_1 > 0$, $\lambda_2 > 0$

The solution of (k_b, ϕ) implies that

$$\begin{aligned}\frac{\partial k_b}{\partial \hat{z}} &= \frac{\partial k_b}{\partial \hat{z}_c} = 1, \\ \frac{\partial \phi}{\partial \hat{z}} &= \frac{\partial \phi}{\partial \hat{z}_c} = \chi f'(k_b), \\ \frac{\partial \phi}{\partial d} &= \delta\left[\chi f'(k_b) - 1\right].\end{aligned}$$

Then

$$\begin{aligned}\frac{\partial \Delta_b(\hat{z}, \hat{z}_c)}{\partial \hat{z}} &= (1-\chi) f'(k_b) - 1, \\ \frac{\partial \Delta_b(\hat{z}, \hat{z}_c)}{\partial \hat{z}_c} &= (1-\chi) f'(k_b) - 1.\end{aligned}$$

In the deposit market, the marginal value of d is

$$\frac{\partial (r_d d)}{\partial d} = \gamma \frac{\partial \phi}{\partial d} = \gamma\delta\left[\chi f'(k_b) - 1\right].$$

Overall, (20) and (21) become

$$\begin{aligned}i &= (2n-1)\left[f'(k_z) - 1\right] + (1-n)\left\{(1-\chi) f'(k_b) - 1 + \gamma\delta\left[\chi f'(k_b) - 1\right]\right\}, \\ s &= (2n-1)\left[f'(k_z) - 1\right] + (1-n)\left[(1-\chi) f'(k_b) - 1\right].\end{aligned}$$

We can derive

$$i - s = (1-n)\gamma\delta\left[\chi f'(k_b) - 1\right]. \tag{31}$$

Similar to the previous case, we obtain coexistence of money and CBDC owing to the binding reserve constraint. A higher i_c makes the LHS of (31), which means k_b would decrease. It then follows that k_z should increase. Notice that $k_b - k_z = \delta\hat{z}$. We know that $\hat{z}$ should decrease and $\hat{z}_c$ would increase as a result of a higher i_c. A higher interest rate of CBDC makes entrepreneurs hold more CBDC and less money. The findings are the same as shown in (30). Therefore, banks take less deposits and issue less loans. Again, a higher i_c on CBDC tends to crowd out bank lending. For type-1 entrepreneurs, unbanked entrepreneurs can purchase more capital but banked entrepreneurs purchase less capital.

In this extension, we allow both money and CBDC to exist in the economy. Interestingly, money and CBDC can coexist only when the reserve constraint binds. In case 1 and case 2 where the reserve constraint does not bind, the marginal benefits of money and CBDC are the same, but CBDC has a lower(higher) marginal cost once $i_c > 0$ ($i_c < 0$). Then the asset with a higher marginal cost will be driven out of existence. Only when $i_c = 0$, money and CBDC can coexist. Once the reserve constraint binds as in case 3 and case 4, money provides additional value as the interest rate earned on deposits depends on the banking fee, which in turn depends on the amount of deposits. If $i_c > 0$, money has a higher marginal cost, but it also has a higher marginal value stemming from the higher interests earn on deposits. Therefore, entrepreneurs are willing to hold a portfolio of money and CBDC when $i_c > 0$. We obtain the coexistence of money and CBDC. While money and CBDC are substitutes in our model, deposits and CBDC become substitutes because banks can only take money as deposits. The model in the extension highlights policymarkers' concern that the introduction of interest-bearing CBDC may crowd out deposits and bank lending. However, our model suggests that a higher interest rate on CBDC does crowd out bank lending, but the overall effect on investment is ambiguous. The higher interest rate on CBDC has a redistribution effect, where unbanked entrepreneurs increase investment and banked entrepreneurs reduce investment.

5 Discussion and Conclusion

To study the effects of introducing CBDC, we firstly build a benchmark model where CBDC exists as the only medium of exchange in the economy. An important feature of CBDC is that the central bank can pay interests on CBDC through digital accounts. The interest rate of CBDC set by the central bank now serves as a new policy tool, in additional to the traditional policy tool where the central bank change the growth rate of CBDC. In our benchmark model, we assess how this additional monetary policy tool affects interest rates, investment and the macroeconomy.

The main findings in the benchmark model is as follows. When the central bank raises the interest rate paid on CBDC, it makes CBDC a more attractive asset to hold. In three out of four cases, we show that an increase in the CBDC interest rate leads to an increase in the amount of CBDC held by entrepreneurs. It follows that entrepreneurs without investment opportunities can deposit more and entrepreneurs with investment opportunities can borrow more. Aggregate investment increases in the economy in response to a higher CBDC interest rate. When neither constraint binds or both constraints bind, we can also show that the higher CBDC interest rate lowers the real deposit rate and the real lending rate. It may seem counterintuitive that a higher CBDC interest rate can reduce the other interest rates. However, notice that the higher CBDC interest rate induces entrepreneurs to hold more CBDC, which increases the amount of deposits and lowers the deposit rate. In addition, it may lead to a reduction of the banking fee or a rise of the amount of loans, both of which contribute to a lower real lending rate.

To consider more CBDC design choices, we also consider another version of the benchmark model (Appendix A), where CBDC and relevant variables are in nominal terms, and more importantly, banks cannot access to CBDC or get paid CBDC interests. The analytical results show that (1) increasing CBDC interest rate usually benefits unbanked type-1 entrepreneurs, in three out of four cases of general equilibrium; (2) the effects on banked entrepreneurs and banking are ambiguous, which may mean more dynamics and robustness from a policy perspective.

Furthermore, we extend the benchmark model by adding paper money to the portfolio holdings of entrepreneurs. This makes us understand more how CBDC interacts with the existing fiat money system. We consider two versions of the extended model: one is based on the benchmark model of "real" CBDC, but now banks only accept cash as deposits but can help store CBDC; the other is based on the benchmark model of "nominal CBDC" (see Appendix B), where banks can accept both cash and CBDC as deposits. The results from the first extended model show that

CBDC and cash can coexist only when the reserve constraint of banks bind (if not, coexisting happens when $i_c = 0$), and increasing i_c leads to a redistribution effect by increasing the investment of unbanked entrepreneurs, but reducing that of banked entrepreneurs. And the main result from the second extended model shows CBDC and cash can coexist only when $i_c = 0$. It is intuititive since two assets can coexist as media of exchange only when they have the same return (zero), or we have to impose some extra conditions to make them coexist.[6] In general, one important message from the extended models is that it is not easy to have cash and interest-bearing CBDC coexist in an economy, and NIR policy is more possible in an economy with a universal CBDC.

CBDC is a frontier research topic, and there is an urgent need to examine many design issues related to it. For example, once issuing CBDC, how should it be issued, through an independent CBDC account system provided by the central bank, or through the current banking infrastructure? This may depend on the size of the economy, and also the banking structure. For example, in China, it may be more feasible to issue CBDC through the current banking infrastructure, since commercial banks really dominate the financial system. Another issue is about who should hold CBDC: individuals? firms? Will there be huge difference when designing CBDC, comparing a corporate finance perspective to a consumer finance perspective? The conjecture is yes, since the deposit and loan markets differentiate so much, for consumers and firms.

It may be normal for individuals or firms to hold CBDC, what about commercial banks or other financial institutions? If banks hold a lot of CBDC, for positive CBDC interest rate, it means they will get paid interests of non-trivial amount. Just recall what commercial banks chose to do in the U.S., after the Fed started to pay interests to bank reserves during the Global Financial Crisis. Instead of making loans as the Fed expected, they chose to hold big-size reserves to get paid the interest, when the

[6] For example, some news said China would firstly experiment introducing CBDC in some area of China, very possible in Shenzhen (a southern Chinese city called "Silicon Valley of China", very close to HongKong) in the near future. This means only banks or firms in this area can access to CBDC.

economy was still bad. Furthermore, for the interest-bearing CBDC, particularly when the interest rate is positive, in the end who should pay for the interests of CBDC? What is the fiscal policy implication? In addition, another important issue is if CBDC should be anonymous or not. If every one hold a digital account of CBDC, naturally the central bank can directly access to the transaction and financial history of each citizen. How to deal with the privacy issues related to this? We know cash transaction is anonymous, which can be good (to protect our privacy), or bad (for the risks of getting stolen and counterfeiting, or using in underground economy). This is also related to data sharing issues in the current era of "digital" economy (Jones and Tonetti, 2018, Easley et al. 2018), but becomes even more important in the context of CBDC design.

There are many interesting issues on CBDC to be explored, and our paper is among the initial attempts to shed light on these issues. We will leave the above questions for future research, particularly from a consumer finance perspective.

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A BM Model II

Now we consider BM Model II, where CBDC is in nominal terms, and banks cannot access to CBDC or get paid CBDC interest. We firstly describe the model environment as follows.

Environment is very similar to BM model in Section 2, except we use nominal terms now. For example, at Stage 1, type-0 entrepreneurs are business/big customers, so bargain with banks about the terms of deposit contract, (d, i_d), where d is amount of deposits, i_d is the nominal deposit rate. The loan market at Stage 2, and the centralized market at Stage 3, are very similar to the BM model. Type-1 entrepreneurs apply for loans in the second market, and bargain with banks about term of loan contract, (ℓ, ϕ), where ℓ is loan size, ϕ is banking service fees.

At Stage 3, for type-0 entrepreneurs in the CM,

$$\begin{aligned} W_0^e(m_c, d) &= \max\{x + \beta \mathbb{E} U^e(\hat{m}_c)\} \\ \text{st. } x + \rho \hat{m}_c &= \rho m_c(1 + i_c) + \rho d(1 + i_d) + \Pi + T. \end{aligned}$$

Then we have,

$$W_0^e(m_c, d) = \rho m_c(1 + i_c) + \rho d(1 + i_d) + \Pi + T + \max_{\hat{m}_c}\{-\rho \hat{m}_c + \beta \mathbb{E} U^e(\hat{m}_c)\}.$$

For type-1 entrepreneurs,

$$\begin{aligned} W_1^e(m_c, \ell, k) &= \max\{x + \beta \mathbb{E} U^e(\hat{m}_c)\} \\ \text{st. } x + \rho \hat{m}_c &= \rho m_c(1 + i_c) - \ell + f(k) + \Pi + T. \end{aligned}$$

Then we have,

$$W_1^e(m_c, \ell, k) = \rho m_c(1 + i_c) - \ell + f(k) + \Pi + T + \max_{\hat{m}_c}\{-\rho \hat{m}_c + \beta \mathbb{E} U^e(\hat{m}_c)\}.$$

And the envelope conditions are,

$$\begin{aligned}\frac{\partial W_0^e(m_c,d)}{\partial m_c} &= \rho(1+i_c), \frac{\partial W_0^e(m_c,d)}{\partial d} = \rho(1+i_d) \\ \frac{\partial W_1^e(m_c,\ell,k)}{\partial m_c} &= \rho(1+i_c), \frac{\partial W_1^e(m_c,\ell,k)}{\partial \ell} = -1, \frac{\partial W_1^e(m_c,\ell,k)}{\partial k} = f'(k).\end{aligned}$$

And FOC against $\hat{m}_c$,

$$\rho = \beta \frac{\partial \mathbb{E} U^e(\hat{m}_c)}{\partial \hat{m}_c}. \tag{32}$$

Then at Stage 1 next period,

$$\mathbb{E}U^e(\hat{m}_c) = nU_1^e(\hat{m}_c) + (1-n)U_0^e(\hat{m}_c), \tag{33}$$

where $U_1^e(\hat{m}_c) = V_1^e(\hat{m}_c)$, since type-1 entrepreneurs will proceed to Stage 2 to apply for bank loans to acquire capital. Since

In the deposit market,

$$\begin{aligned}U_0^e(\hat{m}_c) &= \alpha_0^D W_0^e(\hat{m}_c - d, d) + (1-\alpha_0^D) W_0^e(\hat{m}_c) \\ U^b &= \alpha_b^D V^b(-\hat{\rho} i_d d) + (1-\alpha_b^D) V^b(0) = \alpha_b^D V^b(\omega - \hat{\rho} i_d d)\end{aligned} \tag{34}$$

where $V^b(0) = 0$, since banks getting no deposits will exit from the market. We suppose the matching function is $\mathcal{M}(e_0, b) = \min\{1-n, 1\} = 1-n$. Then $\alpha_0^D = 1, \alpha_b^D = 1-n < 1$. Then the surplus of type-0 entrepreneurs and banks are,

$$\begin{aligned}S_0^D &= \hat{\rho} d(i_d - i_c) \\ S_b^D &= V^b(-\hat{\rho} i_d d) = W^b(\omega - \hat{\rho} i_d d) = \phi - \hat{\rho} i_d d,\end{aligned}$$

where we use the value function $V^b(\omega - \hat{\rho} i_d d)$ at Stage 2, to get the surplus for banks in the deposit market, i.e., $S_b^D = \phi - \hat{\rho} i_d d$.

Suppose the bargaining power of type-0 e is γ, and by Nash bargaining,

$$\max_{d,i_d}[\hat{\rho}d(i_d - i_c)]^{\gamma}[\phi - \hat{\rho}di_d]^{1-\gamma} \tag{35}$$

$$\text{st. } 0 \leq d \leq \hat{m}_c \tag{36}$$

$$i_c \leq i_d < i. \tag{37}$$

At Stage 2, type-1 e apply for bank loans, and with probability α_1^L they get bank loans (we use superscript "b" to represent banked type-1 e), and with the rest probability, they don't (superscript "m" to represent unbanked type-1 e, only using "money", i.e., CBDC here, as internal finance). Correspondingly, the matching rate for banks (those who get deposits at Stage 1) is α_b^L. Suppose the loan contract terms are (p_b, ℓ, ϕ), where p_b is the down payment by CBDC, ℓ is the loan size, and ϕ is banking service fees (all of them are measured by the numaraire goods x). Then, the value function of type-1 e is,

$$\begin{aligned} V_1^e(\hat{m}_c) &= \alpha_1^L[W_1^e(\omega_1 - p_b, k_b) - W_1^e(\omega_1 - p_m, k_m)] + W_1^e(\omega_1 - p_m, k_m) \\ &= \frac{1-n}{n}(\Delta_b - \Delta_m) + W_1^e(\omega_1 - p_m, k_m), \end{aligned} \tag{38}$$

where $\omega_1 \equiv \hat{\rho}\hat{m}_c(1+i_c) - \ell$ ($\ell = 0$ for unbanked type-1 e), $\Delta_b \equiv f(k_b) - k_b - \phi$, and $\Delta_m \equiv f(k_m) - k_m$, p_m is the internal finance by CBDC to pay for k_m, i.e., $p_m = k_m \leq \hat{\rho}\hat{m}_c(1+i_c)$. For banks,

$$\begin{aligned} V^b(\omega - \hat{\rho}di_d) &= W^b(\phi - \hat{\rho}di_d) \\ &= \phi - \hat{\rho}di_d + W^b(0). \end{aligned}$$

Here we use the matching function $\mathcal{M}(e_1, b) = min\{n, 1-n\} = 1-n$ (given $n > 1/2$), then $\alpha_1^L = (1-n)/n$, $\alpha_b^L = 1$.

Suppose the bargaining power of banks for a loan contract is θ, then by Nash bargaining,

$$\max_{\phi,k_b} \phi^\theta [f(k_b) - k_b - \phi - \Delta_m]^{1-\theta} \tag{39}$$

$$\text{st. } k_b - p_b + \phi \leq \chi f(k_b) \tag{40}$$

$$k_b - p_b \leq \delta \hat{\rho} d \tag{41}$$

$$p_b \leq \hat{\rho}\hat{m}_c(1 + i_c). \tag{42}$$

Here we define $\delta \equiv (1/\nu - 1)$, the proportion of reserves against deposits, with ν being the reserve requirements.

A.1 Bargaining Solutions

We sort out the bargaining solutions at the deposit market and loan market in a backward way: firstly solve the bargaining problem at the loan market, then the deposit market. We consider Nash bargaining for two markets, for now.

A.1.1 BS for Loan Market

For the bargaining problem in (39), the down payment constraint (42) shows p_b shall not be greater than all of CBDC type-1 e carry. We know the total amount to acquire k_b should be $q_k k_b = k_b = \ell + p_b$. When the collateral constraint (73) does not bind, p is not uniquely determined, but k_b and ϕ are, hence we choose the solution with the highest p_b, i.e., $p_b = \min\{k^*, \hat{\rho}\hat{m}_c(1+i_c)\}$. Before we sort out the bargaining solutions, we firstly rule out this case: $\hat{\rho}\hat{m}_c(1+i_c) \geq k^*$. In this case, type-1 entrepreneurs do not even need to apply for bank loans, since they already have enough internal finance to acquire the first best k^*, and $\Delta_b = f(k^*) - k^*$. This is not very interesting by economics anyway.

Therefore, we focus on the case $\hat{\rho}\hat{m}_c(1 + i_c) < k^*$, then internal finance and bank credit coexists for banked type-1 e. For simplicity, we suppose the down payment constraint always binds, i.e.,

$$p_b = \hat{\rho}\hat{m}_c(1 + i_c).$$

By the way, for unbanked type-1 e, $p_m \leq \hat{\rho}\hat{m}_c(1 + i_c)$ always binds, then $\Delta_m \equiv f(k_m) - k_m$, and $k_m = p_m = \hat{\rho}\hat{m}_c(1 + i_c)$. And we always have, $k_m < k^*$.

Furthermore, considering the constraints (40) and (41), there are still various cases for banked type-1 e. To sort it out, we construct a Lagrangian function as follows,

$$\begin{aligned}
\mathcal{L}(k_b, \phi, \lambda_1, \lambda_1) &= \phi^{\theta}[f(k_b) - k_b - \phi - \Delta_m]^{1-\theta} - \lambda_1[k_b - p_b + \phi - \chi f(k_b)] \\
&\quad -\lambda_2(k_b - p_b - \delta\hat{\rho}d),
\end{aligned}$$

where $\lambda_j \geq 0$, $j = \{1, 2\}$, λ_1 is the multiplier for the collateral constraint (73) for entrepreneurs, and λ_2 is the multiplier for the reserve constraint (74) for banks. Then the FOCs are as follows,

$$\phi \quad : \quad \frac{\phi^{\theta-1}\{\theta[f(k_b) - k_b - \Delta_m] - \phi\}}{[f(k_b) - k_b - \phi - \Delta_m]^{\theta}} = \lambda_1 \tag{43}$$

$$k_b \quad : \quad \frac{(1-\theta)\phi^{\theta}[f'(k_b) - 1]}{[f(k_b) - k_b - \phi - \Delta_m]^{\theta}} = \lambda_1[1 - \chi f'(k_b)] + \lambda_2 \tag{44}$$

$$\lambda_1 \quad : \quad \lambda_1[k_b - \hat{\rho}\hat{m}_c(1 + i_c) + \phi - \chi f(k_b)] = 0 \tag{45}$$

$$\lambda_2 \quad : \quad \lambda_2[k_b - \hat{\rho}\hat{m}_c(1 + i_c) - \delta\hat{\rho}d] = 0 \tag{46}$$

Since $\lambda_j \geq 0$, by (45) and (46), we need to consider four cases:

Case 1: $\lambda_1 = 0, \lambda_2 = 0$

This is case neither the collateral constraint (40) nor the reserve constraint (41) binds. It means, on one hand, banked type-1 e have enough collateral to get bank credit; on the other hand, banks have enough reserves to make loans at the first best level; then, in the end of the day, type-1 e can pool internal finance p_b and bank credit ℓ to acquire k^*. That is, $k_b = k^*$, where $f'(k^*) = 1$. And from (43), we can derive,

$$\phi = \theta[f(k^*) - k^* - \Delta_m]. \tag{47}$$

Case 2: $\lambda_1 > 0, \lambda_2 = 0$

This is the case the collateral constraint (40) binds, but not the reserve constraint (41). It resembles the scenario that banks have enough reserves to make loans, but entrepreneurs do not have enough collateral to get loans. Hence, in the end of the day, $k_b < k^*$. And by (76)-(79), we have,

$$\begin{aligned}
\phi &= \chi f(k_b) - k_b + \hat{\rho}\hat{m}_c(1+i_c) && (48)\\
\frac{\theta}{1-\theta}\frac{1-\chi f'(k_b)}{(1-\chi)f'(k_b)} &= \frac{\chi f(k_b) - k_b + \hat{\rho}\hat{m}_c(1+i_c)}{(1-\chi)f(k_b) - f(k_m)} && (49)\\
\lambda_1 &= \frac{(1-\theta)\phi^{\theta}[f'(k_b)-1]}{[1-\chi f'(k_b)][f(k_b) - k_b - \phi - \Delta_m]^{\theta}} > 0\\
\lambda_2 &= 0,
\end{aligned}$$

where we can also write λ_1 as $\lambda_1 = \phi^{\theta-1}\{\theta[f(k_b) - k_b - \Delta_m] - \phi\}/[f(k_b) - k_b - \phi - \Delta_m]^{\theta}$.

Case 3: $\lambda_1 = 0, \lambda_2 > 0$

This is the case the collateral constraint (40) does not bind, but the reserve constraint (41) binds. In contrast to Case 2, it means, banks do not have enough reserves to make loans at the first best level, but banked entrepreneurs do have enough collateral to get loans (*if* they can) to pool with internal finance, then acquire the first best k^*. In the end of the day, still $k_b < k^*$. And by (43)-(46), we have,

$$\begin{aligned}
\phi &= \theta[f(k_b) - k_b - \Delta_m] && (50)\\
k_b &= \hat{\rho}\hat{m}_c(1+i_c) + \delta\hat{\rho}d && (51)\\
\lambda_1 &= 0\\
\lambda_2 &= \theta^{\theta}(1-\theta)^{1-\theta}[f'(k_b) - 1] > 0.
\end{aligned}$$

Case 4: $\lambda_1 > 0, \lambda_2 > 0$

This is the case both of the constraints (40) and (41) bind. It is obvious $k_b < k^*$.

And we have,

$$\begin{aligned}
\phi &= \chi f(k_b) - (k_b - p_b) \quad (52)\\
k_b &= p_b + \delta\hat{\rho}d \quad (53)\\
\lambda_1 &= \frac{\phi^{\theta-1}\{\theta[f(k_b) - k_b - \Delta_m] - \phi\}}{[f(k_b) - k_b - \phi - \Delta_m]^{\theta}}\\
\lambda_2 &= \frac{\phi^{\theta-1}\{\phi(1-\theta)(1-\chi)f'(k_b) - \theta[1-\chi f'(k_b)][f(k_b) - k_b - \phi - \Delta_m]\}}{[f(k_b) - k_b - \phi - \Delta_m]^{\theta}}
\end{aligned}$$

A.1.2 BS for Deposit Market

After solving the bargaining problem of the loan market, we are ready to sort out the bargaining solutions for the deposit market (35). For the constraints (36) and (37), given positive surplus for depositors, i.e., $S_0^D > 0$, we only consider the scenario that (36) always binds, i.e., $d = \hat{m}_c$, *and* $i_d > i_c$.

Next we proceed to the bargaining solutions of deposit market. Given $d = \hat{m}_c$, we firstly derive the interior solutions for i_d as follows,

$$i_d = (1-\gamma)i_c + \frac{\gamma\phi}{\hat{\rho}\hat{m}_c} \quad (54)$$

Then, using the bargaining solutions of loan market for case (1)-(4), we can solve i_d for each case.

A.2 General Equilibrium

After sorting out the bargaining solutions for loan market and deposit market, we can define general equilibrium, and solve the whole model. The key is to sort out the asset choice problems in (32).

Using (33), (34), and (38), we have,

$$\begin{aligned}\mathbb{E}U^e(\hat{m}_c) &= (1-n)\hat{\rho}[(\hat{m}_c-d)(1+i_c)+d(1+i_d)]+nk_m \\ &\quad +(2n-1)[f(k_m)-k_m]+(1-n)[f(k_b)-k_b-\phi] \\ &\quad +(1-n)W_0^e(0,0)+nW_1^e(0,0,0),\end{aligned}$$

where $k_m = \hat{\rho}\hat{m}_c(1+i_c)$. We firstly substitute $d = \hat{m}_c$ and (54) to the above, then,

$$\begin{aligned}\mathbb{E}U^e(\hat{m}_c) &= (1-n)\{\hat{\rho}\hat{m}_c[1+(1-\gamma)i_c]+f(k_b)-k_b-(1-\gamma)\phi\} \\ &\quad +(2n-1)[f(k_m)-k_m].\end{aligned}$$

Hence,

$$\begin{aligned}\frac{\partial \mathbb{E}U^e(\hat{m}_c)}{\partial \hat{m}_c} &= \hat{\rho}(1+i_c)\{(2n-1)[f'(k_m)-1]+n\}+\hat{\rho}(1-n)[1+(1-\gamma)i_c] \\ &\quad +(1-n)\{[f'(k_b)-1]\frac{\partial k_b}{\partial \hat{m}_c}-(1-\gamma)\frac{\partial \phi}{\partial \hat{m}_c}\},\end{aligned} \tag{55}$$

where we use $\partial k_m/\partial \hat{m}_c = \hat{\rho}(1+i_c)$. Next, we follow Case (a)-(d) in Section A.1.1, to sort out $\partial k_m/\partial \hat{m}_c$, $\partial k_b/\partial \hat{m}_c$, and $\partial \phi/\partial \hat{m}_c$, for the Euler equations for $\hat{m}_c$ in (32).

Case 1: $\lambda_1 = 0$, $\lambda_2 = 0$

In this case, we can rewrite (32) by substituting bargaining solutions for loan and deposit market, and list all of the GE conditions for $\{k_b, k_m, \phi, i_d, z_c\}$ are as follows,

$$\begin{aligned}k_b &= k^* \\ \phi &= \theta\{f(k^*)-k^*-[f(k_m)-k_m]\} \\ \hat{\rho}d &= \hat{\rho}\hat{m}_c \equiv z_c = k_m/(1+i_c) \\ i_d &= (1-\gamma)i_c+\gamma\theta\frac{f(k^*)-k^*-[f(k_m)-k_m]}{\hat{\rho}\hat{m}_c} \\ 1+i &= (1-n)[1+(1-\gamma)i_c]+(1+i_c)\{B[f'(k_m)-1]+n\},\end{aligned}$$

where $B \equiv 2n-1+\theta(1-n)(1-\gamma) > 0$.

Hence, for the effects of changing i_c, we can derive,

$$\frac{\partial k_m}{\partial i_c} = -\frac{1-\gamma(1-n)+B[f'(k_m)-1]}{A(1+i_c)f''(k_m)} > 0$$
$$\frac{\partial \phi}{\partial i_c} = -\theta[f'(k_m)-1]\frac{\partial k_m}{\partial i_c} < 0$$
$$\frac{\partial z_c}{\partial i_c} = \frac{1}{1+i_c}\frac{\partial k_m}{\partial i_c} - \frac{k_m}{(1+i_c)^2} \lesseqgtr 0$$
$$\frac{\partial i_d}{\partial i_c} = 1-\gamma+\frac{\gamma\phi}{k_m} - \frac{\gamma(1+i_c)}{k_m}\{\phi+\theta k_m[f'(k_m)-1]\}\frac{\partial k_m}{\partial i_c} \lesseqgtr 0$$

Case 2: $\lambda_1 > 0$, $\lambda_2 = 0$

In this case, from (49), we can firstly derive,

$$\frac{\partial k_b}{\partial \hat{m}_c} = (1+i_c)\hat{\rho}\cdot\Omega(k_m,k_b) > 0$$

where $\Omega(k_m,k_b) \equiv \{(1-\theta)(1-\chi)f'(k_b)+\theta[1-\chi f'(k_b)]f'(k_m)\}/\{(1-\chi)f'(k_b)[1-$
$\chi f'(k_b)]$
$-f''(k_b)[(1-\theta)(1-\chi)\phi+\theta\chi((1-\chi)f(k_b)-f(k_m))]\} > 0$. Furthermore, using the above results, we can rewrite (32) as,

$$\begin{aligned} 1+i &= (1+i_c)\{(2n-1)[f'(k_m)-1]+n\}+\gamma(1-n) \\ &\quad (1+i_c)(1-n)[(1-\chi+\gamma\chi)f'(k_b)-\gamma]\Omega(k_m,k_b). \end{aligned}$$

And other GE conditions are as follows,

$$\begin{aligned} \phi &= \chi f(k_b)-k_b+\hat{\rho}\hat{m}_c(1+i_c) \\ \frac{\theta}{1-\theta}\frac{1-\chi f'(k_b)}{(1-\chi)f'(k_b)} &= \frac{\chi f(k_b)-k_b+\hat{\rho}\hat{m}_c(1+i_c)}{(1-\chi)f(k_b)-f(k_m)} \\ \hat{\rho}d &= \hat{\rho}\hat{m}_c \equiv z_c = \frac{k_m}{1+i_c} \\ i_d &= (1-\gamma)i_c+\gamma\frac{\chi f(k_b)-k_b+\hat{\rho}\hat{m}_c(1+i_c)}{\hat{\rho}\hat{m}_c}. \end{aligned}$$

For the effects of changing i_c, there are no clear analytical results for now. We will add calibration for this part later.

Case 3: $\lambda_1 = 0$, $\lambda_2 > 0$

In this case, firstly, substituting the bargaining solution of loan market to (55), then we can rewrite (32) as,

$$\begin{aligned} 1+i &= (1-n)[1+(1-\gamma)i_c] + (1+i_c)\{B[f'(k_m)-1]+n\} \\ &\quad +(1-n)(1+i_c+\delta)[1-\theta(1-\gamma)][f'(k_b)-1] \end{aligned} \tag{56}$$

And the other GE conditions are as follows,

$$\begin{aligned} k_b &= (1+i_c+\delta)\hat{\rho}\hat{m}_c \\ \phi &= \theta\{f(k_b)-k_b-[f(k_m)-k_m]\} \\ \hat{\rho}d &= \hat{\rho}\hat{m}_c \equiv z_c = k_m/(1+i_c) \\ i_d &= (1-\gamma)i_c + \gamma\theta\frac{f(k_b)-k_b-[f(k_m)-k_m]}{\hat{\rho}\hat{m}_c} \end{aligned}$$

Easily, we can derive,

$$k_b = \frac{(1+i_c+\delta)k_m}{(1+i_c)}.$$

Then, we can use the above equation plus (56) for comparative statics,

$$\begin{bmatrix} 1+i_c+\delta & -(1+i_c) \\ B(1+i_c)f''(k_m) & (1-n)(1+i_c+\delta)(1-\theta+\gamma\theta)f''(k_b) \end{bmatrix} \cdot \begin{bmatrix} dk_m \\ dk_b \end{bmatrix} = \begin{bmatrix} k_b - k_m \\ \Psi \end{bmatrix} di_c,$$

where $\Psi = -B[f'(k_m)-1] - n - (1-n)\{1-\gamma+(1-\theta+\gamma\theta)[f'(k_b)-1]\} < 0$. We can easily prove,

$$\begin{aligned} J_c &= \begin{bmatrix} 1+i_c+\delta & -(1+i_c) \\ B(1+i_c)f''(k_m) & (1-n)(1+i_c+\delta)(1-\theta+\gamma\theta)f''(k_b) \end{bmatrix} \\ &= (1-n)(1+i_c+\delta)^2(1-\theta+\gamma\theta)f''(k_b) + B(1+i_c)^2 f''(k_m) < 0. \end{aligned}$$

Since $J_c < 0$, we can prove the existence of GE for Case (3). Furthermore, we can derive,

$$
\begin{aligned}
\frac{\partial k_m}{\partial i_c} &= \frac{(1-n)(1+i_c+\delta)(1-\theta+\gamma\theta)(k_b-k_m)f''(k_b)+(1+i_c)\Psi}{J_c} > 0 \\
\frac{\partial k_b}{\partial i_c} &= \frac{(1+i_c+\delta)\Psi - B(1+i_c)(k_b-k_m)f''(k_m)}{J_c} \\
&\cong B(1+i_c)(k_b-k_m)f''(k_m)-(1+i_c+\delta)\Psi \gtrless 0 \\
\frac{\partial z_c}{\partial i_c} &= \frac{1}{1+i_c}\frac{\partial k_m}{\partial i_c} - \frac{k_m}{(1+i_c)^2} \lesseqgtr 0 \\
\frac{\partial i_d}{\partial i_c} &= 1-\gamma+\gamma\theta\frac{f(k_b)-k_b-[f(k_m)-k_m]}{k_m} + C\frac{\gamma\theta(1+i_c)}{(k_m)^2} \lesseqgtr 0
\end{aligned}
$$

where

$$C = \{[f'(k_b)-1]\frac{\partial k_b}{\partial i_c} - [f'(k_m)-1+f(k_b)-k_b-f(k_m)+k_m]\frac{\partial k_m}{\partial i_c}]\} \lesseqgtr 0.$$

Case 4: $\lambda_1 > 0$, $\lambda_2 > 0$

In this case, the GE conditions for $\{k_b, k_m, \phi, i_d, z_c\}$ are as follows,

$$
\begin{aligned}
\phi &= \chi f(k_b) - \delta\hat{\rho}\hat{m}_c \\
k_b &= (1+i_c+\delta)\hat{\rho}\hat{m}_c \\
\hat{\rho}d &= \hat{\rho}\hat{m}_c \equiv z_c = k_m/(1+i_c) \\
i_d &= (1-\gamma)i_c + \gamma\frac{\chi f(k_b)-\delta\hat{\rho}\hat{m}_c}{\hat{\rho}\hat{m}_c} \\
1+i &= (1+i_c)\{(2n-1)[f'(k_m)-1]+n\}+\gamma(1-n) \\
& \quad (1-n)(1+i_c+\delta)[(1-\chi+\gamma\chi)f'(k_b)-\gamma], \qquad (57)
\end{aligned}
$$

Easily, we can derive,

$$k_b = \frac{(1+i_c+\delta)k_m}{(1+i_c)}. \qquad (58)$$

For the effects of changing i_c, we can use (57) and (58) to get,

$$\begin{bmatrix} 1+i_c+\delta & -(1+i_c) \\ (2n-1)(1+i_c)f''(k_m) & (1-n)(1+i_c+\delta)(1-\chi+\gamma\chi)f''(k_b) \end{bmatrix} \cdot \begin{bmatrix} dk_m \\ dk_b \end{bmatrix} = \begin{bmatrix} k_b - k_m \\ \Phi \end{bmatrix} di_c,$$

where

$$\begin{aligned} \Phi &= -(2n-1)[f'(k_m)-1] - n - (1-n)[(1-\chi+\gamma\chi)f'(k_b)-\gamma] \\ &= -(2n-1)[f'(k_m)-1] - n - (1-n)\{\gamma+(1-\gamma)(1-\chi)]f'(k_b)-\gamma\} \\ &= -(2n-1)[f'(k_m)-1] - n - (1-n)\{\gamma[f'(k_b)-1]+(1-\gamma)(1-\chi)f'(k_b)\} < 0. \end{aligned}$$

And we can also prove,

$$\begin{aligned} J_d &= \begin{bmatrix} 1+i_c+\delta & -(1+i_c) \\ (2n-1)(1+i_c)f''(k_m) & (1-n)(1+i_c+\delta)(1-\chi+\gamma\chi)f''(k_b) \end{bmatrix} \\ &= (1-n)(1+i_c+\delta)^2(1-\chi+\gamma\chi)f''(k_b) + (2n-1)(1+i_c)^2 f''(k_m) < 0. \end{aligned}$$

Since $J_d < 0$, we can prove the existence of GE for Case (d). Furthermore, we can derive,

$$\begin{aligned} \frac{\partial k_m}{\partial i_c} &= \frac{(1-n)(1+i_c+\delta)(1-\chi+\gamma\chi)(k_b-k_m)f''(k_b)+(1+i_c)\Phi}{J_d} > 0 \\ \frac{\partial k_b}{\partial i_c} &= \frac{(1+i_c+\delta)\Phi-(2n-1)(1+i_c)(k_b-k_m)f''(k_b)}{J_d} \\ &\cong (2n-1)(1+i_c)(k_b-k_m)f''(k_b)-(1+i_c+\delta)\Phi \gtreqless 0 \\ \frac{\partial z_c}{\partial i_c} &= \frac{1}{1+i_c}\frac{\partial k_m}{\partial i_c} - \frac{k_m}{(1+i_c)^2} \lesseqgtr 0 \\ \frac{\partial i_d}{\partial i_c} &= 1-\gamma+\frac{\gamma\chi f(k_b)}{k_m} + \gamma\chi\frac{1+i_c}{(k_m)^2}[k_m f'(k_b)\frac{\partial k_b}{\partial i_c} - f(k_b)\frac{\partial k_m}{\partial i_c}] \end{aligned}$$

B Extension II

Now we add cash to BM model II. Suppose entrepreneurs hold a portfolio of cash and CBDC (m_0, m_c) in the beginning of the representative period. From now on, we use the subscripts "0","c" to represent various variables related to cash and CBDC, respectively. Both cash and CBDC are fiat money issued by the central bank in the CM, and can convert to each other at par, so they have the same price ρ, the same growth rate (then the same inflation rate π at steady states). The difference is, m_0 is physical cash which pays 0 return, while m_c is issued to CBDC accounts held by agents in the economy, and pays nominal interest rate i_c every period.[7]

At Stage 3 of the current period, for type-0 entrepreneurs,

$$\begin{aligned} W_0^e(m_0, m_c, d) &= \max_{\hat{m}_0, \hat{m}_c} \{x + \beta \mathbb{E} U^e(\hat{m}_0, \hat{m}_c)\} \\ \text{st. } x + \rho\hat{m}_0 + \rho\hat{m}_c &= \rho m_0 + \rho m_c(1 + i_c) + \rho d(1 + i_d) + \Pi + T. \end{aligned}$$

Then we have,

$$\begin{aligned} W_0^e(m_0, m_c, d) &= \rho m_0 + \rho m_c(1 + i_c) + \rho d(1 + i_d) + \Pi + T \\ &\quad + \max_{\hat{m}_0, \hat{m}_c} \{-\rho\hat{m}_0 - \rho\hat{m}_c + \beta \mathbb{E} U^e(\hat{m}_0, \hat{m}_c)\}. \end{aligned} \tag{59}$$

For type-1 entrepreneurs,

$$\begin{aligned} W_1^e(m_0, m_c, \ell, k) &= \max\{x + \beta \mathbb{E} U^e(\hat{m}_0, \hat{m}_c)\} \\ \text{st. } x + \rho\hat{m}_0 + \rho\hat{m}_c &= \rho m_0 + \rho m_c(1 + i_c) - \ell + f(k) + \Pi + T. \end{aligned}$$

[7] One might question how to justify the coexisting of cash and CBDC for entrepreneurs. It can be, for example, cash comes from sales revenues of entrepreneurs, since a small fraction of consumers may hold only cash, or have no CBDC account, due to economic or technical constraints.

Then we have,

$$W_1^e(m_0, m_c, \ell, k) = \rho m_0 + \rho m_c(1+i_c) - \ell + f(k) + \Pi + T + \max_{\hat{m}_0, \hat{m}_c}\{-\rho\hat{m}_0 - \rho\hat{m}_c + \beta\mathbb{E}U^e(\hat{m}_0, \hat{m}_c)\}. \quad (60)$$

Hence, the envelope conditions are as follows,

$$\frac{\partial W_0^e(m_0, m_c, d)}{\partial m_0} = \frac{\partial W_1^e(m_0, m_c, \ell, k)}{\partial m_0} = \rho$$
$$\frac{\partial W_0^e(m_0, m_c, d)}{\partial m_c} = \frac{\partial W_1^e(m_0, m_c, \ell, k)}{\partial m_c} = \rho(1+i_c)$$
$$\frac{\partial W_0^e(m_0, m_c, d)}{\partial d} = \rho(1+i_d), \frac{\partial W_1^e(m_0, m_c, \ell, k)}{\partial k} = f'(k).$$

And we get the same FOCs for type-0 and type-1 e,

$$\hat{m}_0 : \frac{\rho}{\beta} \geq \frac{\partial\mathbb{E}U^e(\hat{m}_0, \hat{m}_c)}{\partial\hat{m}_0} \quad (61)$$
$$\hat{m}_c : \frac{\rho}{\beta} \geq \frac{\partial\mathbb{E}U^e(\hat{m}_0, \hat{m}_c)}{\partial\hat{m}_c}. \quad (62)$$

When $\hat{m}_0 > 0$, (61) becomes equality; and when $\hat{m}_c > 0$, (62) becomes equality.

For suppliers,

$$W^s(\omega) = \max\{x + \beta U^s\}$$
$$\text{st. } x = \omega + T$$

And at Stage 2,

$$U^s = \max\{-k_b + W^s(\omega + q_k k)\}.$$

Hence, we should have $q_k = 1$ in the competitive capital market. As for banks at

Stage 3,

$$\begin{aligned} W^b(\omega) &= \max\{x+\beta U^b\} \\ \text{st. } x &= \omega + T. \end{aligned}$$

Then, at Stage 1 next period, for entrepreneurs,

$$\mathbb{E}U^e(\hat{m}_0,\hat{m}_c) = nU_1^e(\hat{m}_0,\hat{m}_c) + (1-n)U_0^e(\hat{m}_0,\hat{m}_c). \tag{63}$$

After investment shocks are realized, type-0 e participate in the deposit market, have bilateral meetings and negotiate with banks about deposit contract terms (d_0, d_c, i_d), where d_0, d_c refer to the deposits from cash $(\hat{m}_0)$, CBDC $(\hat{m}_c)$, respectively. Then type-0 e skip Stage 2 and proceed to Stage 3. As for type-1 e, they will skip Stage 1, and directly proceed to Stage 2, i.e., $U_1^e(\hat{m}_0,\hat{m}_c) = V_1^e(\hat{m}_0,\hat{m}_c)$. Hence, for type-0 e,

$$\begin{aligned} U_0^e(\hat{m}_0,\hat{m}_c) &= \alpha_0^D W_0^e(\hat{m}_0 - d_0, \hat{m}_c - d_c, d_0 + d_c, 0) + (1-\alpha_0^D)W_0^e(\hat{m}_0,\hat{m}_c,0,0)] \\ &= \alpha_0^D[\hat{\rho}d_0 i_d + \hat{\rho}d_c(i_d - i_c)] + W_0^e(\hat{m}_0,\hat{m}_c,0,0). \end{aligned} \tag{64}$$

For banks,

$$U^b = \alpha_b^D V^b[-\hat{\rho}(d_0+d_c)i_d] + (1-\alpha_b^D)V^b(0)$$

Since the matching function is $\mathcal{M}(e_0,b) = min\{1-n,1\} = 1-n$, then $\alpha_0^D = 1$, $\alpha_b^D = 1-n < 1$. Suppose only banks that get deposits in the deposit market can survive and proceed to the loan market, then

$$U^b = (1-n)V^b[-\hat{\rho}(d_0+d_c)i_d]$$

Then, the matching surplus for type-0 e and banks are, respectively,

$$S_0^D = \hat{\rho}d_0 i_d + \hat{\rho}d_c(i_d - i_c) \tag{65}$$

$$S_b^D = V^b[-\hat{\rho}(d_0 + d_c)i_d]. \tag{66}$$

Suppose the bargaining power of type-0 e is γ, and by Nash bargaining,

$$\max_{d_0,d_c,i_d} [\hat{\rho}d_0 i_d + \hat{\rho}d_c(i_d - i_c)]^\gamma [V^b(-\hat{\rho}(d_0 + d_c)i_d)]^{1-\gamma} \tag{67}$$

$$\text{st. } 0 < d_0 \leq \hat{m}_0 \tag{68}$$

$$0 \leq d_c \leq \hat{m}_c \tag{69}$$

$$i_d \leq i. \tag{70}$$

At Stage 2, type-1 e apply for bank loans, and with probability α_1^L they get bank loans, and with the rest probability, they don't. Correspondingly, the matching rate for banks (those who get deposits from Stage 1) is α_b^L. Here we again use a special matching function $\mathcal{M}(e_1, b) = min\{n, 1-n\} = 1-n$ (given $n > 1/2$), then $\alpha_1^L = (1-n)/n$, $\alpha_b^L = 1$.

Suppose the loan contract terms are (p_b, ℓ, ϕ), where p_b is the down payment in the form of cash and CBDC, ℓ is the loan size, and ϕ is banking service fees (all of them are measured by the numaraire goods x). Then, the value function of type-1 e is,

$$\begin{aligned} V_1^e(\hat{m}_0, \hat{m}_c) &= \alpha_1^L[W_1^e(\omega_1 - p_b, k_b) - W_1^e(\omega_1 - p_m, k_m)] + W_1^e(\omega_1 - p_m, k_m) \\ &= \frac{1-n}{n}(\Delta_b - \Delta_m) + W_1^e(\omega_1 - p_m, k_m), \end{aligned} \tag{71}$$

where $\omega_1 \equiv \hat{\rho}\hat{m}_0 + \hat{\rho}\hat{m}_c(1+i_c) - \ell$ ($\ell = 0$ for unbanked type-1 e), $\Delta_b \equiv f(k_b) - k_b - \phi$, p_b refers to the down payment by cash and/or CBDC, and $\Delta_m \equiv f(k_m) - p_m$, p_m is the internal finance to pay for k_m, i.e., $p_m = k_m \leq \hat{\rho}\hat{m}_0 + \hat{\rho}\hat{m}_c(1+i_c)$. For banks,

$$\begin{aligned} V^b[\omega - \hat{\rho}(d_0+d_c)i_d] &= W^b[\phi - \hat{\rho}(d_0+d_c)i_d] \\ &= \phi - \hat{\rho}(d_0+d_c)i_d + W^b(0). \end{aligned}$$

Here we again use the matching function $\mathcal{M}(e_1, b) = min\{n, 1-n\} = 1-n$ (given $n > 1/2$), then $\alpha_1^L = (1-n)/n$, $\alpha_b^L = 1$.

Suppose the bargaining power of banks for a loan contract is θ, then by Nash bargaining,

$$\max_{\phi, k_b} \phi^\theta [f(k_b) - k_b - \phi - \Delta_m]^{1-\theta} \tag{72}$$

$$\text{st. } k_b - p_b + \phi \leq \chi f(k_b) \tag{73}$$

$$k_b - p_b \leq \delta\hat{\rho}(d_0 + d_c) \tag{74}$$

$$p_b \leq \hat{\rho}\hat{m}_0 + \hat{\rho}\hat{m}_c(1+i_c). \tag{75}$$

Here we define $\delta \equiv (1/\nu - 1)$, the proportion of reserves against deposits, with ν being the reserve requirements.

B.1 Bargaining Solutions

Now we sort out the bargaining solutions of the deposit market and loan market in a backward way: firstly solve the solutions of the loan market, then the deposit market. We consider Nash bargaining for two markets.

B.1.1 BS for Loan Market

For the bargaining problem in (72), the down payment constraint (75) shows p_b shall not be greater than all of the cash and CBDC type-1 e carry. We know the total amount to acquire k_b should be $q_k k_b = k_b = \ell + p_b$. When the liquidity constraint (73) does not bind, p_b is not uniquely determined, but k_b and ϕ are, hence we choose the solution with the highest p_b, i.e., $p_b = \min\{k_b, \hat{\rho}\hat{m}_0 + \hat{\rho}\hat{m}_c(1+i_c)\}$. Before we sort out the bargaining solutions, we firstly rule out this case: $\hat{\rho}\hat{m}_0 + \hat{\rho}\hat{m}_c(1+i_c) \geq k_b$. In this case, type-1 entrepreneurs do not even need to apply for bank loans, since they already have enough internal finance to acquire the first best k^*, and $\Delta_b = f(k^*) - k^*$. This is not very interesting by economics anyway.

Hence, we focus on the case $\hat{\rho}\hat{m}_0 + \hat{\rho}\hat{m}_c(1+i_c) < k^*$, then internal finance and bank credit coexists for banked type-1 e. For simplicity, we suppose the down payment constraint always binds, i.e.,

$$p_b = \hat{\rho}\hat{m}_0 + \hat{\rho}\hat{m}_c(1+i_c).$$

By the way, for unbanked type-1 e, $p_m \leq \hat{\rho}\hat{m}_0 + \hat{\rho}\hat{m}_c(1+i_c)$ always binds, then $\Delta_m \equiv f(k_m) - k_m$, where $k_m = p_m = \hat{\rho}\hat{m}_0 + \hat{\rho}\hat{m}_c(1+i_c)$. Furthermore, considering the constraints (73) and (74), there are still various cases for banked type-1 e. To sort it out, we construct a Lagrangian function as follows,

$$\begin{aligned}\mathcal{L}(k_b, \phi, \lambda_1, \lambda_1) &= \phi^\theta[f(k_b) - k_b - \phi - \Delta_m]^{1-\theta} - \lambda_1[k_b - p_b + \phi - \chi f(k_b)] \\ &\quad -\lambda_2[k_b - p_b - \delta\hat{\rho}(d_0 + d_c)],\end{aligned}$$

where $\lambda_j \geq 0$, $j = \{1, 2\}$, λ_1 is the multiplier for the collateral constraint (73), and λ_2 is the multiplier for the reserve constraint (74). Then the FOCs are as follows,

$$\phi : \frac{\phi^{\theta-1}\{\theta[f(k_b) - k_b - \Delta_m] - \phi\}}{[f(k_b) - k_b - \phi - \Delta_m]^\theta} = \lambda_1 \tag{76}$$

$$k_b : \frac{(1-\theta)\phi^\theta[f'(k_b) - 1]}{[f(k_b) - k_b - \phi - \Delta_m]^\theta} = \lambda_1[1 - \chi f'(k_b)] + \lambda_2 \tag{77}$$

$$\lambda_1 : \lambda_1\{k_b - [\hat{\rho}\hat{m}_0 + \hat{\rho}\hat{m}_c(1+i_c)] + \phi - \chi f(k_b)\} = 0 \tag{78}$$

$$\lambda_2 : \lambda_2\{k_b - [\hat{\rho}\hat{m}_0 + \hat{\rho}\hat{m}_c(1+i_c)] - \delta\hat{\rho}(d_0 + d_c)\} = 0 \tag{79}$$

Since $\lambda_j \geq 0$, by (78) and (79), we need to consider four cases:

Case 1: $\lambda_1 = 0, \lambda_2 = 0$

This is case neither the liquidity constraint (73) nor the reserve constraint (74) binds. It means, on one hand, banks have enough reserves to make loans at the first best level; on the other hand, banked type-1 e have enough collateral to get bank credit, which can be pooled with internal finance to acquire k^*. That is, $k_b = k^*$. And from (76), we can derive,

$$\phi = \theta[f(k^*) - k^* - \Delta_m]. \tag{80}$$

Case 2: $\lambda_1 > 0, \lambda_2 = 0$

This is the case the liquidity constraint (73) binds, but not the reserve constraint (74). It resembles the scenario that banks have enough reserves to make loans, but entrepreneurs do not have enough collateral to get loans. Hence, again $k_b < k^*$. And by (76)-(79), we have,

$$\begin{aligned}
\phi &= \chi f(k_b) - k_b + [\hat{\rho}\hat{m}_0 + \hat{\rho}\hat{m}_c(1+i_c)] && (81)\\
\frac{\theta}{1-\theta}\frac{1-\chi f'(k_b)}{(1-\chi)f'(k_b)} &= \frac{\chi f(k_b) - k_b + [\hat{\rho}\hat{m}_0 + \hat{\rho}\hat{m}_c(1+i_c)]}{(1-\chi)f(k_b) - f(k_m)} && (82)\\
\lambda_1 &= \frac{(1-\theta)\phi^\theta[f'(k_b)-1]}{[1-\chi f'(k_b)][f(k_b) - k_b - \phi - \Delta_m]^\theta} > 0\\
\lambda_2 &= 0,
\end{aligned}$$

where we can also write λ_1 as $\lambda_1 = \phi^{\theta-1}\{\theta[f(k_b) - k_b - \Delta_m] - \phi\}/[f(k_b) - k_b - \phi - \Delta_m]^\theta$.

Case 3: $\lambda_1 = 0, \lambda_2 > 0$

This is the case the liquidity constraint (73) does not bind, but the reserve constraint (74) binds. In contrast to Case 2, it means, banks do not have enough reserves to make loans at the first best level, but banked entrepreneurs do have enough collateral to get loans (*if* they can) to pool with internal finance, then acquire the first best k^*. In the end of the day, still $k_b < k^*$. And by (76)-(79), we have,

$$\begin{aligned} \phi &= \theta[f(k_b) - k_b - \Delta_m] && (83) \\ k_b &= \hat{\rho}\hat{m}_0 + \hat{\rho}\hat{m}_c(1+i_c) + \delta\hat{\rho}(d_0 + d_c) && (84) \\ \lambda_1 &= 0 \\ \lambda_2 &= \theta^\theta(1-\theta)^{1-\theta}[f'(k_b) - 1] > 0. \end{aligned}$$

Case 4: $\lambda_1 > 0, \lambda_2 > 0$

This is the case both of the constraints (73) and (74) bind. It is obvious $k_b < k^*$. And we have,

$$\phi = \chi f(k_b) - \delta\hat{\rho}(d_0 + d_c) \tag{85}$$

$$k_b = \hat{\rho}\hat{m}_0 + \hat{\rho}\hat{m}_c(1+i_c) + \delta\hat{\rho}(d_0 + d_c) \tag{86}$$

$$\lambda_1 = \frac{\phi^{\theta-1}\{\theta[f(k_b) - k_b - \Delta_m] - \phi\}}{[f(k_b) - k_b - \phi - \Delta_m]^\theta}$$

$$\lambda_2 = \frac{\phi^{\theta-1}}{[f(k_b) - k_b - \phi - \Delta_m]^\theta}\{\phi(1-\theta)(1-\chi)f'(k_b) \\ -\theta[1 - \chi f'(k_b)][f(k_b) - k_b - \phi - \Delta_m]\}$$

B.1.2 BS for Deposit Market

After solving the bargaining problem of the loan market, we are ready to sort out the bargaining solutions for the deposit market. For the deposit constraints (68) and (69), given positive inflation rate, it makes sense that (68) always binds, i.e., $d_0 = \hat{m}_0$. As for the constraint (69), it will depend the relative values of i_d and i_c, which we discuss as follows.

Let us begin from an unusual case, $i_d < i_c$. Obviously, in this case, type-0 e would not be willing to deposit CBDC at banks. Furthermore, they would have no motivation to deposit cash as well, since they can convert cash to CBDC at par, and earn the same interest rate $i_c > i_d$. Therefore, to have $S_0^D > 0$, it must satisfy $i_d \geq i_c$.

Notice that, if $i_d = i_c$, type-0 e will be indifferent to hold CBDC or bank deposits.

Next we proceed to the bargaining solutions of deposit market. After the above discussion about different scenarios for i_d and i_c, it is without loss of generality that we suppose both of the constraints (68) and (69) bind, i.e., $d_0 = \hat{m}_0$, $d_c = \hat{m}_c$. It means, when $i_d \geq i_c$, type-0 e are willing to deposit both cash and CBDC at banks (at least being indifferent between CBDC and deposits). Furthermore, we can solve the solution for i_d as,

$$i_d = \frac{(1-\gamma)\hat{\rho}\hat{m}_c}{\hat{\rho}(\hat{m}_0 + \hat{m}_c)} i_c + \frac{\gamma\phi}{\hat{\rho}(\hat{m}_0 + \hat{m}_c)}. \tag{87}$$

Since we already derive ϕ from the bargaining solutions for the loan market (Case 1-4), we can derive i_d for the four cases as well.

B.2 General Equilibrium

After sorting out the bargaining solutions for loan market and deposit market, we can define general equilibrium, and solve the full model. The key is to sort out the asset choice problems in (59) or (60).

From (63), (64) and (71), we have,

$$\begin{aligned} \mathbb{E}U^e(\hat{m}_0, \hat{m}_c) &= (2n-1)[f(k_m) - k_m] + (1-n)[f(k_b) - k_b - (1-\gamma)\phi] + \\ &\quad k_m - \gamma(1-n)\hat{\rho}\hat{m}_c i_c + (1-n)W_0^e(0,0,0,0) + nW_1^e(0,0,0,0), \end{aligned}$$

where we use $d_0 = \hat{m}_0$, $d_c = \hat{m}_c$ and (87), and $k_m = \hat{\rho}\hat{m}_0 + \hat{\rho}\hat{m}_c(1 + i_c)$.

Then,

$$
\begin{aligned}
\frac{\partial \mathbb{E}U^e(\hat{m}_0,\hat{m}_c)}{\partial \hat{m}_0} &= \hat{\rho}\{1+(2n-1)[f'(k_m)-1]\}+ \\
&\quad (1-n)\{[f'(k_b)-1]\frac{\partial k_b}{\partial \hat{m}_0}-(1-\gamma)\frac{\partial \phi}{\partial \hat{m}_0}\} \\
\frac{\partial \mathbb{E}U^e(\hat{m}_0,\hat{m}_c)}{\partial \hat{m}_c} &= \hat{\rho}(1+i_c)\{1+(2n-1)[f'(k_m)-1]\}-\gamma(1-n)\hat{\rho}i_c+ \\
&\quad (1-n)\{[f'(k_b)-1]\frac{\partial k_b}{\partial \hat{m}_c}-(1-\gamma)\frac{\partial \phi}{\partial \hat{m}_c}\}.
\end{aligned}
$$

Next, we follow Case 1-4 in Section B.1.1, to sort out $\partial k_b/\partial \hat{m}_0$, $\partial k_b/\partial \hat{m}_c$, $\partial \phi/\partial \hat{m}_0$ and $\partial \phi/\partial \hat{m}_c$, for the Euler equations for $\hat{m}_0$ and $\hat{m}_c$ in (61) and (62).

Case 1: $\lambda_1=0$, $\lambda_2=0$

In this case, with $k_b=k^*$, and ϕ in (80), we have,

$$
\begin{aligned}
\frac{\partial k_b}{\partial \hat{m}_0} &= \frac{\partial k_b}{\partial \hat{m}_c}=0 \\
\frac{\partial \phi}{\partial \hat{m}_0} &= -\theta\hat{\rho}[f'(k_m)-1] \\
\frac{\partial \phi}{\partial \hat{m}_c} &= -\theta\hat{\rho}(1+i_c)[f'(k_m)-1].
\end{aligned}
$$

Then we can rewrite (61) and (62) as,

$$
\begin{aligned}
i &\geqslant B[f'(k_m)-1] \\
\frac{i-i_c}{1+i_c} &\geqslant B[f'(k_m)-1]-\frac{\gamma(1-n)i_c}{1+i_c},
\end{aligned}
$$

where $B\equiv(2n-1)+\theta(1-n)(1-\gamma)>0$. As mentioned before, when $\hat{m}_0>0, \hat{m}_c>0$, the above Euler equations becomes "equality". But we need to check if $\hat{m}_0>0, \hat{m}_c>0$ can be satisfied at the same time, i.e., if cash and CBDC can coexist in this economy. There are three scenarios as follows:

Subcase (i): $i_c>0$. With a positive CBDC interest rate, agents are not willing to hold cash, or choose to convert all cash to interest-bearing CBDC, i.e., $m_0=0$. Hence, the whole economy will move to a universal CBDC system, as in BM Model II.

And we will get the same results for the new policy tool, i.e., changing i_c, as in Case 1 of Appendix A. Once in a universal CBDC system, it is possible to have $i_c < 0$.

Subcase (ii): $i_c < 0$. With a negative CBDC interest rate, no agents are willing to hold CBDC, or choose to convert all CBDC to cash, i.e., $m_c = 0$. Hence, the whole economy will move to a pure paper money system. The new policy tool of changing i_c will not be available.

Subcase (iii): $i_c = 0$. Since CBDC has the same zero return as cash, agents are indifferent to hold cash or CBDC. This can be the case that cash can coexist with CBDC. But the new policy tool of changing i_c will not be available.

Case 2: $\lambda_1 > 0$, $\lambda_2 = 0$

In this case, using (82) we have,

$$\begin{aligned} \frac{\partial k_b}{\partial \hat{m}_0} &= \hat{\rho}\Omega(k_m, k_b) > 0 \\ \frac{\partial k_b}{\partial \hat{m}_c} &= \hat{\rho}(1+i_c)\Omega(k_m, k_b) > 0, \end{aligned}$$

where $\Omega(k_m, k_b) > 0$ is the same as in Case 2 of BM Model II. Furthermore, by (81) and (82), we can rewrite (61) and (62) as,

$$\begin{aligned} i &\geq (2n-1)[f'(k_m)-1] + (1-n)\{\Omega(k_m,k_b)[f'(k_b)-1+(1-\gamma)(1-\chi f'(k_b))] - (1-\gamma)\} \\ \frac{i-i_c}{1+i_c} &\geqslant (2n-1)[f'(k_m)-1] + (1-n)\{\Omega(k_m,k_b)[f'(k_b)-1+(1-\gamma)(1-\chi f'(k_b))] \\ &\quad -(1-\gamma)\} - \frac{\gamma(1-n)i_c}{1+i_c}. \end{aligned}$$

Similarly, we need to check if $\hat{m}_0 > 0, \hat{m}_c > 0$ can be satisfied at the same time for the above Euler equations, and need to consider three subcases as in **Case 1**.

Case 3: $\lambda_1 = 0$, $\lambda_2 > 0$

With (83) and (84), we have,

$$\begin{aligned}
\frac{\partial k_b}{\partial \hat{m}_0} &= \hat{\rho}(1+\delta) \\
\frac{\partial k_b}{\partial \hat{m}_c} &= \hat{\rho}(1+\delta+i_c) \\
\frac{\partial \phi}{\partial \hat{m}_0} &= \theta\hat{\rho}\{(1+\delta)[f'(k_b)-1]-[f'(k_m)-1]\} \\
\frac{\partial \phi}{\partial \hat{m}_c} &= \theta\hat{\rho}\{[f'(k_b)-1](1+i_c+\delta)-[f'(k_m)-1](1+i_c)\}.
\end{aligned}$$

Then, we can rewrite (61) and (62) as,

$$\begin{aligned}
i &\geq B[f'(k_m)-1]+(1-n)[1-\theta(1-\gamma)][f'(k_b)-1](1+\delta) \\
\frac{i-i_c}{1+i_c} &\geqslant B[f'(k_m)-1]+(1-n)[1-\theta(1-\gamma)][f'(k_b)-1](1+\frac{\delta}{1+i_c})-\frac{\gamma(1-n)i_c}{1+i_c}.
\end{aligned}$$

Similarly, we need to check if $\hat{m}_0 > 0, \hat{m}_c > 0$ can be satisfied at the same time for the above Euler equations, and need to consider three subcases as in **Case 1**.

Case 4: $\lambda_1 > 0$, $\lambda_2 > 0$

With (85) and (86), we have,

$$\begin{aligned}
\frac{\partial k_b}{\partial \hat{m}_0} &= \hat{\rho}(1+\delta) \\
\frac{\partial k_b}{\partial \hat{m}_c} &= \hat{\rho}(1+\delta+i_c) \\
\frac{\partial \phi}{\partial \hat{m}_0} &= \hat{\rho}[\chi f'(k_b)(1+\delta)-\delta] \\
\frac{\partial \phi}{\partial \hat{m}_c} &= \hat{\rho}[\chi f'(k_b)(1+\delta+i_c)-\delta].
\end{aligned}$$

Hence, we can rewrite (61) and (62) as,

$$\begin{aligned}
i &\geq (2n-1)[f'(k_m)-1]+(1-n)\{(1+\delta)[(1-(1-\gamma)\chi)f'(k_b)-1]+\delta(1-\gamma)\} \\
\frac{i-i_c}{1+i_c} &\geqslant (2n-1)[f'(k_m)-1]+(1-n)\{[(1+\frac{\delta}{1+i_c})[(1-(1-\gamma)\chi)f'(k_b)-1]+\frac{\delta(1-\gamma)}{1+i_c}\} \\
&\quad -\frac{\gamma(1-n)i_c}{1+i_c}.
\end{aligned}$$

Similarly, we need to check if $\hat{m}_0 > 0, \hat{m}_c > 0$ can be satisfied at the same time for the above Euler equations, and need to consider three subcases as in **Case 1**.